Report from Dagstuhl Seminar 25041

Solving Problems on Graphs: From Structure to Algorithms

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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 25041 "Solving Problems on Graphs: From Structure to Algorithms", which was held from 19 January to 24 January 2025. The report contains abstracts for presentations about recent structural and algorithmic developments for a variety of graph problems. It also contains a collection of open problems which were posed during the seminar.

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1 Executive Summary

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Many discrete optimization problems can be modeled as graph problems, leading to a long list of well-studied problems, which include graph partitioning, covering and packing problems, network design problems, width parameter problems, and so on. Most of these graph problems are computationally hard. However, this situation may change if we require the input to belong to some special graph class. This leads to two fundamental questions, which formed the focus of our Dagstuhl Seminar: for which classes of graphs can a computationally hard graph problem be solved in polynomial time, and for which classes of graphs does the problem remain hard?

One of the main research aims of our seminar was to discover new insights that lead to results for a whole range of problems rather than just for a single problem alone. That is, our goal was to determine general properties, such that large classes of graph problems sharing certain common features can be solved efficiently on a graph class if and only if the graph

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class has these properties. For this purpose, our seminar brought together researchers from Discrete Mathematics, working in structural graph theory, and researchers from Theoretical Computer Science, working in algorithmic graph theory. In total, 41 participants from 13 different countries attended the seminar.

The scientific program of the seminar consisted of 18 sessions: 5 survey talks of fifty minutes, 10 contributed talks of at most thirty minutes, and 5 open problem sessions. This left ample time for discussions and problem-solving. Participants presented the progress that was made during the seminar during several "progress report" sessions.

Each of the five survey talks covered a particular structural or algorithmic key aspect of the seminar in order to enable collaborations of researchers with different backgrounds. On Monday, Tuukka Korhonen presented a survey on new results on induced minors of graphs. This is a well-known graph containment relation, but recently developed techniques led to significant progress and new open problems. On the same day, Édouard Bonnet discussed a number of new results and open problems on twin-width, a relatively new graph width parameter that is being studied intensively. The final survey talk on Monday was given by Stefan Kratsch, who discussed a number of open problems around modular-based graph parameters and graph width parameters from the perspective of parameterized complexity.

On Tuesday, Maria Chudnovsky presented an overview of the induced subgraphs and tree decompositions project that currently comprises a series of 18 papers. Afterward, Nicolas Trotignon explained the importance of layered wheels. These are graphs of arbitrarily large treewidth that play a significant role in the induced subgraphs and tree decompositions project. In general, a layered wheel consists of many paths, all pairwise with edges between them (thus guaranteeing large treewidth), where many additional desired properties may be forced. Thus layered wheels provide a useful source of boundary examples of families of graphs with large treewidth. A few weeks after the completion of our seminar, three of the participants, Bogdan Alecu, Édouard Bonnet, and Nicolas Trotignon, found, together with Pedro Bureo Villafana, a large new family of layered wheels providing counterexamples to several known conjectures in the field, some of which were discussed at the seminar.

The five general open problem sessions took place on Monday, Tuesday, Wednesday, and Thursday. Details of the presented problems can be found in the report, together with abstracts of all the talks.

We thank Julien Codsi for his help with the Dagstuhl Report of our seminar.

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3 Overview of Talks

3.1 Twin-Width: Algorithmic Applications and Open Questions

Édouard Bonnet (ENS – Lyon, FR)

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We review some algorithmic applications of the relatively new twin-width parameter, and propose several open problems.

3.2 Forbidding induced subgraphs: structure and algorithms

Maria Chudnovsky (Princeton University, US)

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Tree decompositions are a powerful tool in both structural graph theory and graph algorithms. Many hard problems become tractable if the input graph is known to have a tree decomposition of bounded "width". Exhibiting a particular kind of a tree decomposition is also a useful way to describe the structure of a graph.

Tree decompositions have traditionally been used in the context of forbidden graph minors; studying them in connection with graph containment relations of more local flavor (such as induced subgraph or induced minors) is a relatively new research direction. In this talk we will discuss recent progress in this area, touching on both the classical notion of bounded tree-width, and concepts of more structural flavor.

In particular we will describe a recent result providing a complete list of induced subgraph obstructions to bounded pathwidth.

3.3 Tree independence number of graphs with no induced $S_{t,t,t}$, $K_{t,t}$ and line graph of subdivided walls

Julien Codsi (Princeton University, US), Maria Chudnovsky (Princeton University, US), Daniel Lokshtanov (University of California – Santa Barbara, US), Martin Milanič (University of Primorska, SI)

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URL https://doi.org/10.48550/ARXIV.2501.14658

The tree independence number is a natural generalization of treewidth, offering versatility for a wide range of algorithmic applications. In this talk, we explore the reasoning behind a recent result that establishes a polylogarithmic bound on the tree independence number for graphs excluding certain structures: induced $K_{t,t}, S_{t,t,t}$ ($K_{1,3}$ where each edge is subdivided $t-1$ times), and the line graph of a subdivided large wall. Consequently, this graph class admits quasipolynomial-time algorithms for numerous problems that are typically NP-Hard. Beyond its algorithmic implications, our method provides valuable structural insights, particularly into the identification of desirable balanced separators within this graph class.

3.4 Bandwidth is FPT-Approximable

Daniel Lokshtanov (University of California – Santa Barbara, US), Maria Chudnovsky (Princeton University, US)

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Joint work of Daniel Lokshtanov, Eran Nevo, Maria Chudnovsky

A linear layout of a graph G is a bijection $f : V(G) \to 1...n$, and the bandwidth of a linear layout f is the maximum over all edges uv of G of $|f(u) - f(v)|$. The bandwidth $bw(G)$ of a graph G is the minimum bandwidth of a linear layout of G.

Computing the bandwidth of an input graph G is a notoriously hard problem: it is NP-hard to approximate within any constant factor [1] and W[t] hard for every t [2] , and these hardness results carry over even to very restricted sub-classes of trees.

We prove that bandwidth is FPT-approximable: there exists an algorithm that takes as input a graph G and integer k, runs in time $f(k)n^{O(1)}$ and either concludes that the bandwidth of G is more than k, or produces a layout with bandwidth at most $g(k)$.

On the way we show the following structural result: there exists a function h, such that for every graph G and integer k, either G contains a sub-tree of bandwidth at least k, or the bandwidth of G is at most $h(k)$.

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3.5 Graphs of bounded sphere dimension

Meike Hatzel (IBS – Daejeon, KR)

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Joint work of Meike Hatzel, James Davies, Agelos Georgakopoulos, Rose McCarty

The *sphere dimension* of a graph G is the smallest integer $d \geq 2$ so that G is an intersection graph of metric spheres in $\mathbb{R}^d$. This talk considers the class $\mathcal{C}^d$ of graphs with sphere dimension d. We present the results that for each integer t, the class of all graphs in $\mathcal{C}^d$ that exclude $K_{t,t}$ as a subgraph has strongly sublinear separators and that $\mathcal{C}^d$ has asymptotic dimension at most $2d + 2$.

3.6 Minimal obstructions to C_5-coloring in hereditary graph classes

Jorik Jooken (KU Leuven, BE), Jan Goedgebeur (KU Leuven, BE), Paweł Rzążewski (Warsaw University of Technology, PL), and Oliver Schaudt (Bayer AG – Leverkusen, DE)

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Joint work of Jorik Jooken, Jan Goedgebeur, Karolina Okrasa, Paweł Rzążewski, Oliver Schaudt
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URL https://doi.org/10.4230/LIPICS.MFCS.2024.55

For graphs G and H, an H-coloring of G is an edge-preserving mapping from $V(G)$ to $V(H)$. Note that if H is the triangle, then H-colorings are equivalent to 3-colorings. In this paper we are interested in the case that H is the five-vertex cycle C_5.

A minimal obstruction to C_5-coloring is a graph that does not have a C_5-coloring, but every proper induced subgraph thereof has a C_5-coloring. In this paper we are interested in minimal obstructions to C_5-coloring in F-free graphs, i.e., graphs that exclude some fixed graph F as an induced subgraph. Let P_t denote the path on t vertices, and let $S_{a,b,c}$ denote the graph obtained from paths $P_{a+1}, P_{b+1}, P_{c+1}$ by identifying one of their endvertices.

We show that there is only a finite number of minimal obstructions to C_5-coloring among F-free graphs, where $F \in \{P_8, S_{2,2,1}, S_{3,1,1}\}$ and explicitly determine all such obstructions. This extends the results of Kamiński and Pstrucha [1] who proved that there is only a finite number of P_7-free minimal obstructions to C_5-coloring, and of Dębski et al. [2] who showed that the triangle is the unique $S_{2,1,1}$-free minimal obstruction to C_5-coloring.

We complement our results with a construction of an infinite family of minimal obstructions to C_5-coloring, which are simultaneously P_{13}-free and $S_{2,2,2}$-free. We also discuss infinite families of F-free minimal obstructions to H-coloring for other graphs H.

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3.7 Induced minors of graphs

Tuukka Korhonen (University of Copenhagen, DK)

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The theory of induced minors of graphs aims to generalize the theory of graph minors to dense graph classes. This dates back to the end of the 80s and the start of the 90s, when several negative results in this direction were proven. However, in the recent years there has been a resurgence of interest into induced minors, and some positive results and interesting conjectures have been discovered. In this talk I survey the theory of induced minors and the most interesting open problems in this area, focusing mostly on the algorithmic perspective.

3.8 Graph structure and parameterized algorithms

Stefan Kratsch (HU Berlin, DE)

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Joint work of Narek Bojikian, Vera Chekan, Falko Hegerfeld, Stefan Kratsch, Pascal Kunz

The talk surveys some recent work on parameterized algorithms for graph problems relative to structural parameters: (1) Efficient parameterized algorithms that obtain in polynomial time a approximate solution with additive error depending on a chosen graph parameter. (2) Key ideas behind the recent tight bound for Steiner Tree parameterized by clique-width. (3) Tight bounds relative to multi-clique-width.

3.9 Restricted CSPs and H-free Algorithmics

Barnaby Martin (Durham University, GB)

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Joint work of Barnaby Martin, Santiago Guzman-Pro

In recent years, much attention has be placed on the complexity of graph homomorphism problems when the input is restricted to P_k-(subgraph)-free graphs. We consider the directed version of this research line, by addressing the question: is it true that digraph homomorphism problems CSP(H) have a P versus NP-complete dichotomy when the input is restricted to DP_k-(subgraph)-free digraphs (where DP_k is the directed path on k vertices)? We build on the theory of constraint satisfaction problems to address this question.

Our first main results are a P versus NP-complete classification of CSPs when the input is restricted to $\mathcal{F}$-homomorphism-free digraphs, or restricted to CSP(H') for some finite digraph H'. We then use the established connection to constraint satisfaction theory to show our third main result (and partial answer to the question above): if CSP(H) is NP-complete, then there is a positive integer N such that CSP(H) remains NP-hard even for DP_N-(subgraph)-free digraphs. Moreover, CSP(H) remains NP-hard for DP_N-(subgraph)-free acyclic digraphs, and becomes polynomial-time solvable for DP_{N-1}-(subgraph)-free acyclic digraphs.

Another contribution of this work is verifying the question above for digraphs on three vertices and a family of smooth tournaments.

3.10 The Complexity of Diameter on H-free graphs

Jelle Oostveen (Utrecht University, NL)

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Main reference Jelle J. Oostveen, Daniël Paulusma, Erik Jan van Leeuwen: "The Complexity of Diameter on H-free Graphs", in Proc. of the Graph-Theoretic Concepts in Computer Science – 50th International Workshop, WG 2024, Gozd Martuljek, Slovenia, June 19-21, 2024, Revised Selected Papers, Lecture Notes in Computer Science, Vol. 14760, pp. 444–459, Springer, 2024.
URL https://doi.org/10.1007/978-3-031-75409-8_31

The intensively studied DIAMETER problem is to find the diameter of a given connected graph. We investigate, for the first time in a structured manner, the complexity of DIAMETER for H-free graphs, that is, graphs that do not contain a fixed graph H as an induced subgraph. We first show that if H is not a linear forest with small components, then DIAMETER cannot be solved in subquadratic time for H-free graphs under SETH. For some small linear forests, we do show linear-time algorithms for solving DIAMETER. For other linear forests H, we make progress towards linear-time algorithms by considering specific diameter values. If H is a linear forest, the maximum value of the diameter of any graph in a connected H-free graph class is some constant $d_{\max}$ dependent only on H. We give linear-time algorithms for deciding if a connected H-free graph has diameter $d_{\max}$, for several linear forests H. In contrast, for one such linear forest H, DIAMETER cannot be solved in subquadratic time for H-free graphs under SETH. Moreover, we even show that, for several other linear forests H, one cannot decide in subquadratic time if a connected H-free graph has diameter $d_{\max}$ under SETH.

3.11 Partial Grundy Coloring is FPT

Fahad Panolan (University of Leeds, GB)

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Main reference Akanksha Agrawal, Daniel Lokshtanov, Fahad Panolan, Saket Saurabh, Shaily Verma: "Parameterized Saga of First-Fit and Last-Fit Coloring", in Proc. of the 42nd International Symposium on Theoretical Aspects of Computer Science, STACS 2025, March 4-7, 2025, Jena, Germany, LIPIcs, Vol. 327, pp. 5:1–5:21, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2025.
URL https://doi.org/10.4230/LIPICS.STACS.2025.5

The classic greedy coloring algorithm considers the vertices of an input graph G in a given order and assigns the first available color to each vertex v in G. In the Grundy Coloring problem, the task is to find an ordering of the vertices that will force the greedy algorithm to use as many colors as possible. In the Partial Grundy Coloring, the task is also to color the graph using as many colors as possible. This time, however, we may select both the ordering in which the vertices are considered and which color to assign the vertex. The only constraint is that the color assigned to a vertex v is a color previously used for another vertex if such a color is available. Aboulker et al. [1] proved that Grundy Coloring is W[1]-hard. In this talk, we prove that Partial Grundy Coloring is fixed-parameter tractable.

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3.12 Erdős-Pósa property of tripods in directed graphs

Michal Pilipczuk (University of Warsaw, PL), Meike Hatzel (IBS – Daejeon, KR)

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Joint work of Michal Pilipczuk, Meike Hatzel, Karolina Okrasa, Marcin Briański
Main reference Marcin Brianski, Meike Hatzel, Karolina Okrasa, Michal Pilipczuk: "Erdös-Pósa property of tripods in directed graphs", CoRR, Vol. abs/2408.16733, 2024.
URL https://doi.org/10.48550/ARXIV.2408.16733

In a directed graph D with sources S and sinks T, a tripod is the union of a pair of $S - T$ paths that have different sources but the same sink. We prove that tripods in directed graphs have the Erdős-Pósa property. The key element in the proof is the use of the Matroid Intersection Theorem. We will also mention open problems about possible generalizations and related algorithmic problems.

3.13 Maximum k-colorable Induced Subgraph (and beyond) in Polynomial Time

Roohani Sharma (MPI für Informatik – Saarbrücken, DE)

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URL https://doi.org/10.1137/1.9781611977912.189
Main reference Daniel Lokshtanov, Paweł Rzążewski, Saket Saurabh, Roohani Sharma, Meirav Zehavi: "Maximum Partial List H-Coloring on P_5-free graphs in polynomial time", CoRR, Vol. abs/2410.21569, 2024.
URL https://doi.org/10.48550/ARXIV.2410.21569

In this talk we report a successful resolution of an open problem from the previous edition (Dagstuhl Seminar 22481) of this seminar.

At Dagstuhl Seminar 22481, Michal Pilipczuk asked whether one can find a maximum 2-colorable induced subgraph on P_5-free graphs in polynomial time or even in quasi-polynomial time.

We [Agrawal, Lima, Lokshtanov, Saurabh, Sharma] designed a quasi-polynomial time algorithm for the problem, which appeared at SODA 2024 [1]. Later together with Rzążewski, we improved the running time to polynomial for the same problem. This work will appear at ACM TALG 2025.

In a follow-up work [2], we [Lokshtanov, Rzążewski, Saurabh, Sharma, Zehavi] show that one can also design a polynomial time algorithm on P_5-free graphs for an even more general problem: the Maximum k-Colorable Induced Subgraph problem, for any fixed positive integer k. This can be further generalised to the Maximum Weight Partial List H-Coloring problem.

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3.14 A survey of layered wheels

Nicolas Trotignon (CNRS – Ecole Normale Supérieure de Lyon, FR)

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Layered wheels are constructions of graphs first discovered by Ni Luh Dewi Sintiari and the speaker. They disprove several conjectures and answer several questions. For instance, jointly with Maria Chudnovsky, the speaker could use them to disprove a conjecture de Dallard, Milanič and Štorgel about the tree-independence number and a conjecture of Hajebi about the induced subgraphs contained in graphs of high treewidth. The goal of the talk is to present all the situations where layered wheels turned out to be useful and to present in more detail the variant that disproved the two conjectures mentioned above.

3.15 Weighted Graph Problems and Protrusions

Michal Wlodarczyk (University of Warsaw, PL)

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Main reference Michal Wlodarczyk: "Losing Treewidth In The Presence Of Weights", in Proc. of the 2025 Annual ACM-SIAM Symposium on Discrete Algorithms, SODA 2025, New Orleans, LA, USA, January 12-15, 2025, pp. 3743–3761, SIAM, 2025.
URL https://doi.org/10.1137/1.9781611978322.125

I will talk about vertex-deletion problems that become harder when the vertices are weighted. This is because we do not know a weighted counterpart of the technique known as "protrusion replacement". I will talk about a recent work on approximation for Weighted Treewidth-η Deletion which circumvents this issue. Then I will move on to some open problems related to weighted graphs and protrusions.

4 Open problems

4.1 Hardness/tractability radius of Hamiltonian Cycle

Édouard Bonnet (ENS – Lyon, FR)

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Main reference Édouard Bonnet: "Answering Related Questions", CoRR, Vol. abs/2501.10633, 2025.
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Given a problem Π, a distance dist over its inputs, and a function $d : N \to \mathbb{R}_+ \cup \{\infty\}$, $Sidestep(\Pi, dist, d)$ is the computational problem that takes an input I of Π, and is required to output a pair $(J, \Pi(J))$ where $dist(I, J) \leq d(|I|)$, and $\Pi(J)$ is a correct answer to Π on input J. This framework is introduced and motivated in [1].

We give one example of an open question in this setting (more are given in the preprint). $Sidestep(HamiltonianCycle, dist_e, n/3)$ is polynomial-time solvable, where $dist_e$ is the edge edit distance (i.e., the minimum number of edges to edit to go from one graph to the other) while $Sidestep(HamiltonianCycle, dist_e, n^{1/2-o(1)})$ is NP-hard. What is the "smallest" d such that $Sidestep(HamiltonianCycle, dist_e, d)$ is tractable?

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1 Édouard Bonnet. *Answering Related Questions*. (2025), Preprint available on arxiv `https://arxiv.org/abs/2501.10633`

4.2 Polylogarithmic bounds on treewidth

Maria Chudnovsky (Princeton University, US)

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For every positive integer t there exists an integer c_t with the following property. Let G be a graph with no complete subgraph of size t, and no induced minor isomorphic to $K_{t,t}$ is the txt wall. Then the treewidth of G is bounded from above by $c_t \log^{c_t} |V(G)|$.

4.3 Atoms vs. Avoiding Simplicial Vertices

Konrad Dabrowski (Newcastle University, GB)

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Main reference Karl Boddy, Konrad K. Dabrowski, Daniël Paulusma: "Atoms Versus Avoiding Simplicial Vertices", in Proc. of the Algorithms and Complexity – 14th International Conference, CIAC 2025, Rome, Italy, June 10-12, 2025, Proceedings, Part II, Lecture Notes in Computer Science, Vol. 15680, pp. 299–315, Springer, 2025.
URL https://doi.org/10.1007/978-3-031-92935-9_19

A clique cut-set in a connected graph is a clique whose deletion results in a disconnected graph. A graph is an atom if it has no clique cut-set. A simplicial vertex is one whose neighbourhood is a clique. If an atom is not a complete graph, then it cannot contain any simplicial vertices.

For a hereditary graph class $\mathcal{G}$, we denote its set of connected graphs by $\mathcal{G}^c$, its set of atoms by $\mathcal{G}^c_a$, and its set of connected graphs that are either an atom or else have no simplicial vertices by $\mathcal{G}^c_s$. Equivalently, $\mathcal{G}^c_s$ consists of all connected graphs that are either complete or have no simplicial vertices. Note that $\mathcal{G}^c_a \subseteq \mathcal{G}^c_s \subseteq \mathcal{G}^c$. A hereditary graph class $\mathcal{G}$ will have $\mathcal{G}^c_a \neq \mathcal{G}^c_s$ if and only if G contains at least one graph from one of 27 infinite families of minimal graphs that have no simplicial vertices, but are not atoms. Roughly speaking, the graphs in these families consist of exactly two holes, which are linked together by a clique cut-set that has size at most 4 and that may contain some vertices of the two holes.

There are many computational problems that are polynomial-time solvable on a hereditary graph class $\mathcal{G}$ whenever they are polynomial-time solvable on the atoms in $\mathcal{G}^c_a$. If we replace $\mathcal{G}^c_a$ with $\mathcal{G}^c_s$ in the previous sentence, does the statement apply to a strictly wider range of computational problems?

More formally, does there exist a natural graph problem Π, for which the following holds:

1. for every hereditary graph class $\mathcal{G}$, the problem Π is polynomial-time solvable on $\mathcal{G}$ if and only if it is polynomial-time solvable on $\mathcal{G}^c_s$, and
2. there exists a hereditary graph class $\mathcal{F}$ such that Π is NP-hard on $\mathcal{F}$, but polynomial-time solvable on $\mathcal{F}^c_a$.

4.4 Counting Graphs with Vertex Cover of Size k

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Given as input two positive integers n and k, compute the number of graphs with vertex set $\{1, \ldots, n\}$ and at least one vertex cover of size at most k. The naive algorithm does this in time 2^{n^2}. Does there exist an algorithm with running time $2^{o(n^2)}$?

4.5 Longest Cycle Problem Parameterized Above Combinatorial Bounds

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The task of the classical Longest Cycle problem is, given a graph G and a positive integer k, to decide whether G has a cycle of length at least k. This problem generalizes Hamiltonian Cycle and is well-known to be NP-complete. From the positive side, Longest Cycle is known to be FPT when parameterized by k, and there is a long history of research focused on developing various algorithmic tools for the problem. On the other hand, various lower bounds on the circumference of a graph, i.e., the length of the longest cycle, are well-known in Extremal Combinatorics. This raises the question of whether it is possible to combine the strengths of both areas. Recently [4, 5], Longest Cycle was investigated for the parameterization above the classical circumference lower bounds of Dirac [2] and Erdős and Gallai [3], respectively. These results are obtained for undirected graphs and the area of directed graphs is completely unexplored. This leads to a plethora of open problems. For example, any directed graph of minimum in-degree $\delta^+(G) \geq 1$ has a cycle of length at least $\delta^+(G) + 1$. Is it possible to find a cycle of length $\delta^+(G) + k$ in FPT in k time on (2-strong) digraphs? More generally, we ask whether it is possible to obtain interesting and nontrivial parameterized algorithms for Longest Cycle on directed graphs above combinatorial bounds and refer to the survey of Bermond and Thomassen [1] for the circumference bounds.

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4.6 Single-exponential time algorithms parameterized by clique-width without k-expression

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When a k-expression witnessing that a graph has clique-width at most k is given, many NP-hard graph problems can be solved in time $2^{O(k)}n^{O(1)}$. Can we obtain such algorithms parameterized by clique-width without the assumption that a k-expression is given as an input? Oum, Sæther, and Vatshelle gave $2^{O(k \log k)}n^{O(1)}$ time algorithms in [1].

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4.7 The complexity of k-Induced Disjoint Paths on H_3-subgraph-free graphs

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Let H_i be the graph obtained from two copies of P_3 with a path of length i joining the central vertices. For k, fixed, the k-Induced Disjoint Paths problem takes as input a graph with k terminal pairs $(s_1, t_1), \ldots, (s_k, t_k)$ and asks if there are vertex-disjoint paths connecting the terminal pairs such that there are no edges between these paths.

It is known that k-Induced Disjoint Paths is in P for H_1-subgraph-free graphs and for H_2-subgraph-free graphs. Furthermore, it is NP-complete for H_i-subgraph-free graphs for all $i \geq 4$ [1]. What is the complexity in the H_3-subgraph-free case?

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4.8 Feedback vertex set (and similar problems) in P_t-free bipartite graphs

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(k, ϕ)-Maximum Weight Induced Subgraph is a meta-problem that asks for a maximum-weight induced subgraph of treewidth at most k that models an CMSO2 formula ϕ; this problem captures Maximum Independent Set and Feedback Vertex Set, among others. It is known to be solvable in quasi-polynomial time in P_t-free graphs [2] and we conjecture it is actually polynomial-time solvable in these graph classes; the conjecture is confirmed up to $t = 6$ [1].

A recent manuscript [3] established this conjecture in P_7-free graphs of bounded clique number. Somewhat surprisingly, the case of P_7-free bipartite graphs turned out to be an important subcase.

This motivates the following question: can we establish polynomial-time solvability of (k, ϕ)-Maximum Weight Induced Subgraph in P_t-free bipartite graphs? A special case of Feedback Vertex Set is already very interesting. (Note that Maximum Independent Set is polynomial-time solvable in bipartite graphs thanks to the maximum matching techniques.)

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4.9 Detecting almost complete induced minors

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For some fixed graph H, one may wonder whether detecting H as an induced minor can be done in polynomial time. For some H's, the problem is known to be polytime solvable, for others it is known to be NP-complete, so that a dichotomy theorem would be good, and this is a well-known open question that is widely open. A direction might therefore be to find graph some graphs H that are likely to give insight on the general goal. I propose $H = K_t \setminus e$, that is the graph obtained from the complete graph on t vertices by removing one edge.

These are very "close" to be polytime to detect because K_t is. Indeed, containing K_t as an induced minor is equivalent to containing it as a minor. So, to detect K_t, one may rely on the classical Robertson and Seymour algorithm to detect a minor. Having $H = K_t \setminus e$ being NP-complete would therefore be striking, but not unlikely since adding a non-edge constraint is really demanding a lot. On the other hand, it might be polytime by some clever reduction to detect a minor, or by a structure theorem for $K_t \setminus e$-induced-minor-free graphs, or by some other mean.

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