Report from Dagstuhl Seminar 25281

From Sparse Interpolation to Signal Processing: New Synergies

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Abstract

In a data-rich digital world, finding sparse, efficient representations – especially for multi-exponential models – has become critical, particularly when measurements are costly or noisy. These models, which involve complex or real exponents, underpin key processes in signal processing, relaxation dynamics, chemical reactions, heat transfer, and fluid dynamics, with widespread real-world impact. The challenge lies at the intersection of several computational disciplines: structured matrices, rational approximation, sparse interpolation, quadrature, tensor decompositions, and subdivision methods – each offering potential pathways to more robust and efficient algorithms. Multi-exponential analysis is foundational across engineering and industry, enabling advances in DOA estimation, remote sensing, MRI, superresolution, seismology, radio astronomy, and telecommunications – areas vital to energy, health, transportation, and space research. This Dagstuhl Seminar "From Sparse Interpolation to Signal Processing: New Synergies" (25281) brought together experts from computational harmonic analysis, numerical linear algebra, computer algebra, signal processing, approximation theory, and engineering applications to foster cross-disciplinary collaboration and accelerate innovation in this dynamic field.

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1 Executive Summary

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In today's data-driven world, where vast volumes of information are generated across scientific, medical, and technological domains, the challenge of extracting meaningful insights from limited, noisy, or high-dimensional measurements has become more pressing than ever. A central problem in this context is the identification of sparse, low-complexity model representations – specifically, models that can accurately describe complex phenomena using the fewest possible parameters or measurements. This is particularly critical in the analysis of multi-exponential signals, where the goal is to recover a small number of exponential components (with real or complex exponents) from a minimal set of observations – often referred to as "probes" or "samples". The acquisition of such measurements is frequently constrained by practical limitations. In medical imaging, for instance, scanning time directly impacts patient comfort and throughput. In remote sensing or radio astronomy, data collection involves expensive instrumentation and limited bandwidth. In industrial testing, measurement costs can be prohibitive. As a result, researchers are increasingly forced to work with datasets that are not only limited in size but also corrupted by noise, making accurate reconstruction a non-trivial task. **This has elevated the need for robust, efficient, and theoretically grounded methods for exponential analysis – methods that can deliver high-fidelity results even under severe data scarcity and noise contamination.** Multi-exponential analysis, though seemingly abstract, plays a surprisingly central role in numerous everyday technologies and scientific disciplines. At its core, it involves decomposing a signal into a sum of exponentials – functions of the form

$$f(t) = \sum_{k=1}^{r} c_k \, \mathrm{e}^{\lambda_k t},$$

where c_k and λ_k may be real or complex.

When the exponents are complex, such models are essential in analyzing oscillatory signals – common in digital signal processing, time series forecasting, and spectral estimation. These techniques are used to extract frequencies, damping rates, and amplitudes from noisy data, forming the backbone of applications ranging from audio and speech processing to financial market modeling and biomedical signal analysis.

When the exponents are real, multi-exponential models describe fundamental physical processes: relaxation dynamics in materials science, decay rates in radioactive isotopes, reaction kinetics in chemistry, heat diffusion in engineering systems, and fluid flow in environmental modeling. These models are not just theoretical constructs – they are indispensable tools for understanding and predicting real-world behavior across physics, biology, and engineering.

The mathematical and computational challenges of multi-exponential analysis are deeply intertwined with several advanced areas of computational science. The problem naturally connects to **structured matrix theory** – where the Hankel or Vandermonde structure of the data matrix encodes the exponential form – enabling efficient algorithms via rank minimization and low-rank approximation. **Rational approximation theory** provides powerful tools for modeling the Z-transformed signal data as a ratio of polynomials, thereby linking the λ_k and c_k to poles and residues. **Sparse interpolation techniques** allow for the recovery of parameters from few samples, while scale-and-shift invariance principles offer robustness to signal transformations. **Tensor decomposition methods** and **multivariate quadrature** extend these ideas to multi-dimensional data, where the curse of dimensionality poses a fundamental challenge, and advanced techniques including sparsity models are of high importance. Furthermore, **non-convex optimization** plays a crucial role, as the parameter estimation problem often leads to non-convex cost functions with multiple local minima – requiring sophisticated initialization and convergence strategies. **Subdivision methods** further extend the reach of these techniques into geometric and directional data analysis.

The impact of multi-exponential analysis extends far beyond theory. It is foundational in a wide array of **engineering and industrial applications**: direction-of-arrival (DOA) estimation in radar and wireless communications, high-resolution remote sensing and satellite imaging, antenna array design for 5G and beyond, digital image reconstruction and superresolution, precision metrology in manufacturing, radio astronomy for detecting faint cosmic signals, and magnetic resonance imaging (MRI), where fast, accurate reconstruction enables shorter scan times and improved diagnostics. These technologies are not only advancing scientific discovery but are also addressing major societal challenges – improving healthcare outcomes, enabling sustainable energy systems, enhancing transportation safety, supporting space exploration, and strengthening global communication networks.

Given this broad relevance, this Dagstuhl Seminar "From Sparse Interpolation to Signal Processing: New Synergies" (25281) aimed to serve as a vital interdisciplinary forum, bringing together experts from computational harmonic analysis, numerical linear algebra, computer algebra, nonlinear approximation theory, digital signal processing, as well as partners from industry. By fostering dialogue between researchers who have developed similar concepts in isolation, we hope to catalyze cross-fertilization, unify methodologies, and identify shared challenges and opportunities. The goal has been not only to advance the theoretical foundations of exponential analysis but also to accelerate the development of next-generation algorithms that are faster, more robust, and scalable – ultimately enabling breakthroughs in data science, engineering, and beyond.

The talks of this Seminar have been organized with emphasis to the following 6 main topics:

- **Generalisations of exponential analysis**
  (G. Plonka-Hoch, Y. Segman, H. Mhaskar, R. O'Dowd, D. Potts, and J. Prestin),
- **Exponential analysis and structured matrices**
  (A. Matos, H. Liang, M. Ishteva, T. Sauer, and A. Iske)
- **Exponential analysis in computational science**
  (W.-S. Lee, J. Gielis, D. Li, A. Beutler, and R. Beinert)
- **Exponential analysis in quadrature and subdivision**
  (A. Cuyt, T. Perez, M. Cotronei, and M. Piñar)
- **Exponential analysis in engineering**
  (D. de Villiers, D. Davidson, J. Gilmore, N. Diab, and A. Terui)
- **Exponential analysis and computer algebra**
  (J. Gerhard, E. Kaltofen, B. Grenet, and P. Giorgi)

The seminar began on Monday with overview lectures on the first five main topics of the meeting, enabling all participants to quickly gain an entry into the fascinating open interdisciplinary challenges surrounding exponential analysis.

The talk topics on Tuesday covered connections between exponential analysis and sparse approximation, structured matrices, as well as problems in signal analysis and signal separation.

On Wednesday, the topics focused on multivariate integration and subdivision. The free afternoon was used for a short trip to Bernkastel-Kues by several participants and provided the opportunity for physical exercise and lively discussions on the seminar topics.

Thursday was dedicated to various application-oriented topics in exponential analysis. The discussions ranged from sparse models in biology and confocal microscopy to questions concerning the efficient measurement of mutual coupling terms in linear arrays.

Finally, the close connections to special problems in computer algebra, as for example, a quasi-linear time sparse interpolation algorithm over the integers or the fast interpolation and multiplication of unbalanced polynomials, have been the main topic on the final day.

We would like to highlight that this seminar builds upon the 2015 Dagstuhl Seminar 15251, titled “Sparse Modelling and Multi-Exponential Analysis” and the 2022 Dagstuhl Seminar 22221 “Exponential Analysis: Theoretical Progress and Technological Innovation”.

The discussions held during the 2015 event sparked numerous fruitful collaborations, including the successful Horizon 2020 RISE project EXPOWER – short for “Exponential Analysis Empowering Innovation” (Grant Agreement No. 101008231, running from 2021-2026), with Annie Cuyt as the coordinator. This project exemplifies how foundational research in exponential analysis can translate into impactful, cross-sectoral innovation.

In October 2025, three months after this meeting, we submitted a new proposal to the Call: Horizon-MSCA-2025-SE-01 (MSCA Staff Exchanges 20225) with the topic TRESUR – “Building synergies between industry and mathematical topics in sparse approximation and recovery”, coordinated by Tereza Pérez, in close collaboration with A. Cuyt, W.-S. Lee, A. Matos, M. Piñar, D. de Villiers, G. Plonka-Hoch and several further participants of this Dagstuhl Seminar.

Our experience confirms that Dagstuhl Seminars serve as timely and transformative forums for scientific exchange. They create fertile ground for new partnerships, stimulate interdisciplinary thinking, and unlock novel research directions. In light of rapid advances in both theoretical methods and real-world applications, there is a growing need to strengthen the bridge between cutting-edge mathematical developments and practical industrial challenges. This seminar, and our ongoing efforts, aim to foster such connections – ensuring that theoretical progress continues to inspire and inform real-world innovation.

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3 Generalizations of Exponential Analysis

3.1 Prony's Method and Generalizations

Gerlind Plonka (University of Göttingen, DE, plonka@math.uni-goettingen.de)

Joint work of Gerlind Plonka, Kilian Stampfer

The generalized Prony method introduced in [1] is a reconstruction technique for a large variety of sparse signal models that can be represented as sparse expansions into eigen functions of a linear operator A. However, this procedure requires the evaluation of higher powers of the linear operator A that are often expensive to provide. In this survey talk we propose two important extensions of the generalized Prony method that simplify the acquisition of the needed samples essentially and at the same time can improve the numerical stability of the method. The first extension regards the change of operators from A to $\phi(A)$, where ϕ is a suitable operator valued mapping, such that A and $\phi(A)$ possess the same set of eigenfunctions. The goal is now to choose ϕ such that the powers of $\phi(A)$ are much simpler to evaluate than the powers of A. The second extension concerns the choice of the sampling functionals. We show, how new sets of different sampling functionals F_k can be applied with the goal to reduce the needed number of powers of the operator A (resp. $\phi(A)$) in the sampling scheme and to simplify the acquisition process for the recovery method.

This talk is based on joint work with Kilian Stampfer, see [2].

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3.2 Structure Aware Matrix Pencil Method

Yehonatan Segman (Technion – Israel Institute of Technology, Haifa, IL, yehonatans@campus.technion.ac.il)

We address the problem of detecting the number of complex exponentials and estimating their parameters from a noisy signal using the Matrix Pencil (MP) method. We introduce the MP modes and present their informative spectral structure. We show theoretically that these modes can be divided into signal and noise modes, where the signal modes exhibit a perturbed Vandermonde structure. Leveraging this structure, we proposed a new MP algorithm, termed the SAMP algorithm, which has two novel components. First, we present a new and robust model order detection with theoretical guarantees. Second, we present an efficient estimation of signal amplitudes. We show empirically that the SAMP algorithm significantly outperforms the standard MP method, particularly in challenging conditions with closely spaced frequencies and low Signal-to-Noise Ratio (SNR) values. Additionally, compared with prevalent information based criteria, we show that SAMP is more computationally efficient and insensitive to noise distribution.

3.3 Blind Source Signal Separation Using Localized Kernels

Hrushikesh Mhaskar (Claremont Graduate University, US, Hrushikesh.Mhaskar@cgu.edu)

Joint work of Eric Mason, Hrushikesh Mhaskar, Sippanon Kitimoon

The task of separating a superposition of signals into its individual components is a common challenge encountered in various signal processing applications, especially in domains such as audio and radar signals. A previous paper by Chui and Mhaskar [1] proposes a method called Signal Separation Operator (SSO) to find the instantaneous frequencies and amplitudes of such superpositions where both of these change continuously and slowly over time. In this talk, we amplify and modify this method in order to separate linear chirp signals in the presence of crossovers, a very low SNR, and discontinuities.

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3.4 A Signal Separation View of Classification

Ryan O'Dowd (Claremont Graduate University, US, ryan.o'dowd@cgu.edu)

Joint work of Hrushikesh Mhaskar, Ryan O'Dowd

The main problem of signal separation is to determine the locations ω_k of a signal of the form $\mu(t) = \sum_{k=1}^{K} a_k\, \delta_{\omega_k}(t)$, given observations

$$\widetilde{\mu}(j) = \widehat{\mu}(j) + \varepsilon(j) = \sum_{k=1}^{K} a_k e^{i\omega_k j} + \varepsilon(j), \tag{1}$$

where the $\varepsilon(j)$'s are independent samples from some noise distribution. A key piece of information is the minimal separation among the ω_k's, which dictates the complexity of a model necessary to recuperate the locations to a given accuracy. As this minimal separation tends to zero, we end up in a regime known as super-resolution, which poses its own challenges.

In this work we examine a generalization of the signal separation and super-resolution settings by considering a measure of the form

$$\mu(t) = \sum_{k=1}^{K} c_k\, \mu_k(t),$$

where μ_k's are each measures on some unknown domain. By allowing the constituent measures themselves to be supported on some dense set, our problem of interest incorporates both signal separation and super-resolution as particular cases. We give theory and a method to estimate the supports of the μ_k's given only finitely many samples from μ.

One application of this work lies in machine learning, where we have developed an algorithm for data classification in the active learning paradigm. Therefore, we are able to view signal separation, super-resolution, and machine learning classification problems under a unified umbrella.

3.5 Operator Learning and Sparse Approximation

Daniel Potts (Technische Universität Chemnitz, DE, potts@mathematik.tu-chemnitz.de)

In this talk, we present algorithms for the approximation of multivariate functions. We start with the approximation by trigonometric polynomials based on sampling of multivariate functions on rank-1 lattices. To this end, we study the approximation of functions in periodic Sobolev spaces of dominating mixed smoothness. The proposed algorithm based mainly on a one-dimensional fast Fourier transform, and the arithmetic complexity of the algorithm depends only on the cardinality of the support of the trigonometric polynomial in the frequency domain. After a detailed introduction we will focus on the following questions in more detail.

- We discuss methods where the support of the trigonometric polynomial is unknown.
- We describes an extension of approximation methods for nonperiodic functions via a multivariate change of variables.
- Based on these methods we develop algorithms for discrete operator learning.

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3.6 Sparse Interpolation of Multivariate Functions of Bounded Variation

Jürgen Prestin (University of Lübeck, DE, juergen.prestin@uni-luebeck.de)

Joint work of Jürgen Prestin, E. Semenova

This talk deals with the approximation error of trigonometric interpolation for multivariate functions of bounded variation in the sense of Hardy-Krause. We propose interpolation operators related to both the tensor product and sparse grids on the multivariate torus. For these interpolation processes, we investigate the corresponding error estimates in the Lp norm for the class of functions under consideration. In addition, we compare the accuracy with the cardinality of these grids in both approaches. This is joint work with E. Semenova (Institute of Mathematics, Ukrainian Academy of Sciences, Kiev and University of Lübeck).

4 Structured Matrices

4.1 Structured Matrices and Rational Approximation

Ana Matos (University of Lille, FR, Ana.Matos@univ-lille.fr)

After showing the link between exponential sums models, structured matrices and rational approximation, we will present some results concerning two problems studied within the work package 2 of the EXPOWER project.

The first problem concerns stability problems when leading with Hankel matrix pencils. Given a signal $f(t) = \sum_{j=1}^{r} \alpha_j \exp(\phi_j t), \quad$ with $\alpha_j, \phi_j \in \mathbb{C}$, the aim is to recover the values of the coefficients $\alpha_j,\ j = 1 \cdots r$ and the (mutually distinct) exponents $\phi_j,\ j = 1 \cdots r$. The problem reduces to the computation of eigenvalues of a Hankel pencil,

$$H_n^{(1)} v_j = \lambda_j H_n^{(0)} v_j, \ \ H_r^{(m)} = (f_{m+i+j-2})_{i,j=1}^{r}, \ \ \lambda_j = \exp(\phi_j \Delta),$$

$$f_k = f(k\Delta),\ k = 0, \cdots, 2r-1$$

where the sampling interval Δ satisfies the Shannon-Nyquist criteria. Starting from a singular pencil $(\widetilde{H}^{(0)}, \widetilde{H}^{(1)})$ polluted by noise of size ϵ, we need to project it into adequate subspaces in order to obtain a regular pencil and then do a perturbation analysis. We obtain upper bounds on the chordal distances between the perturbed and exact eigenvalues and obtain in this way the sensitivity of the eigenvalues. We also get bounds on the euclidean relative error of the corresponding eigenvectors .

The second problem concerns rational approximation and model reduction. From a rational matrix function of type $(N-1, N)$, $H(s) = C(sE - A)^{-1}B + D$, where C, E, A, B are matrices, we are looking for computing recursively strictly proper rational matrix functions H_n of size $m_1 \times m_2$ with Mc-Millan degree $\leq n$, $H_n(s) = C_n(sE_n - A_n)^{-1}B_n + D_n$ satisfying some tangential interpolation conditions. We obtained a formula for the linearized error, and we propose an AAA-type algorithm to compute a sequence of approximants $H_n(s)$ satisfying some tangential interpolation conditions and some error optimization criteria. This is a work in progress.

4.2 Unlabeled Sensing Using Rank-One Moment Matrix Completion

Hao Liang (Chinese Academy of Sciences, Beijing, CN, lianghao2020@amss.ac.cn)

We study the unlabeled sensing problem that aims to solve a linear system of equations $Ax = \pi(y)$ for an unknown permutation π. For a generic matrix A and a generic vector y, we construct a system of polynomial equations whose unique solution satisfies $A\xi^* = \pi(y)$. In particular, ξ^* can be recovered by solving the rank-one moment matrix completion problem. We propose symbolic and numeric algorithms to compute the unique solution. Some numerical experiments are conducted to show the efficiency and robustness of the proposed algorithms.

4.3 Solving Systems of Polynomial Equations with Tensors

Mariya Ishteva (KU Leuven, BE, mariya.ishteva@kuleuven.be)

Joint work of Mariya Ishteva, Philippe Dreesen

Solving systems of polynomial equations is a fundamental problem, with applications in (applied) mathematics, science, and engineering. Although different approaches have been considered in the literature, the problem remains difficult.

Our solution strategy is based on tensor techniques. We first build a partially symmetric tensor from the coefficients of the polynomials. The factors of its (partially symmetric) canonical polyadic decomposition can then be used for building systems of linear equations, which reveal the solutions of the original system.

Future work includes comparisons with existing methods and extending the class of problems, for which the method can be applied.

4.4 Inverses of Multivariate Hankel Matrices

Tomas Sauer (University of Passau, DE & Fraunhofer IIS, DE, Tomas.Sauer@uni-passau.de)

Inverses of Hankel matrices can be given in a somewhat explicit way by means of the so-called Bezoutians. The talk gives the corresponding result for a nonsingular multivariate Hankel matrix, i.e., a matrix formed in the canonical way from a multiindexed moment sequence. It turns out that one obtains a formula for each coordinate direction and that all these formulas involve the orthogonal polynomials as well as the monic H-basis for the associated Prony ideal. This clearly highlights the intimate connection of the problem to exponential polynomials and their reconstruction from equispaced samples.

4.5 A Refined Ingham-Type Theorem for Spectral Properties of Kernel Matrices

Armin Iske (University of Hamburg, DE, armin.iske@uni-hamburg.de)

Joint work of Armin Iske, Tizian Wenzel

We discuss a refined multivariate Ingham-type theorem, whereby we obtain localisation estimates for integrals of exponential sums from symbol functions. This allows us to improve on previous results concerning spectral properties of kernel matrices, including estimates for their spectral condition number. We finally place particular emphasis on spectral alignment for pairs of kernels with finite but different smoothness. This talk is based on joint work with Tizian Wenzel (LMU Munich).

5 Exponential Analysis in Computational Science

5.1 Exponential Analysis Applications

Wen-shin Lee (University of Stirling, GB, wen-shin.lee@stir.ac.uk)

The Blahut/Ben-Or/Tiwari black-box sparse polynomial interpolation algorithm from computer algebra is closely related to exponential analysis, which in turn connects to various areas of computational science. Its links with structured matrix theory, rational approximation and tensor decomposition have opened new possibilities to improve numerical algorithms that are fundamental to a wide range of engineering and industrial applications. These include fluorescence-lifetime imaging microscopy (FLIM), direction-of-arrival (DOA) estimation, remote sensing, antenna design, radar imaging, super-resolution, testing and metrology, radio astronomy, magnetic resonance imaging (MRI), seismology, and financial time series analysis. We report on several recent developments in these areas. Such applications have the potential to address major societal challenges in energy, transportation, space research, healthcare, and telecommunications.

5.2 Inequality and Diversity: Insights from Biology

Johan Gielis (Genicap Beheer BV – Tilburg, NL, johan.gielis@gmail.com)

Inequality is an important topic in economics and human society. In biology similar phenomena are found at the level of individual, organizations and ecosystems. It seems however, that the mathematics underlying these phenomena is very similar, and a new field of research, Econobiology, evolves. Here, we focus on our results from biology and mathematical formulations. The generalized performance equations (product exponentials) are examined in relation to the Lorenz curve, an important tool for measuring income inequality in economics. The relationship between the graphs is provided by 135° rotating and shifted Lorenz curve. This transformation is named Shi Rotations. The results show that the advanced performance models provide an excellent fit for all models tested (leaves in bamboo, melon fruits, tepals of Magnolia flowers and diversity in forests). The Gini coefficient used in economics of inequality is turns out to be closely related to the coefficient of variation, and to other indices such as the Theil index and generalized entropy index. This should lead to new ways of studying nature and human societies, from a dynamical, not a static viewpoint. Although inequality and diversity are two sides of the same coin, this should be done with caution.

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5.3 Compact Non-Invasive Cerebral Blood Flow Sensing

David Li (University of Strathclyde – Glasgow, GB, David.Li@strath.ac.uk)

Continuous, non-invasive monitoring of cerebral blood flow (CBF) is critically important for managing patients in intensive care, particularly neonates and individuals suffering from stroke or traumatic brain injury. Traditional imaging modalities such as MRI or CT remain impractical for bedside or long-term monitoring due to their cost, size, and the need for patient transport. Dr Li's team has developed a compact, low-cost CBF sensing platform based on near-infrared diffuse correlation spectroscopy (DCS), integrated with advanced single-photon avalanche diode (SPAD) sensors. This system offers high sensitivity to microvascular blood flow dynamics with minimal hardware complexity. While current single- or dual-channel systems can monitor CBF changes at discrete sites, the team's vision is to extend this to a multi-channel platform capable of reconstructing three-dimensional images of cerebral perfusion in real time. However, this transition from localised sensing to volumetric imaging poses significant computational and mathematical challenges, especially in addressing the ill-posed inverse problem associated with reconstructing CBF maps from sparse, noisy data. In this presentation, Dr Li will first provide an overview of the physiological motivations for non-invasive, continuous CBF monitoring and outline the limitations of existing technologies. He will then introduce the principles behind DCS and SPAD-based detection, and share results from their current system, including real-time CBF signal recovery in human subjects. The core focus will be on their roadmap towards a multi-channel CBF tomography system, combining hardware innovation with advanced image reconstruction algorithms. The team is particularly keen to engage with mathematicians and computational scientists specialising in inverse problems, compressed sensing, and sparse sampling. Their aim is to identify robust, efficient reconstruction methods to deliver accurate 3D CBF distributions from a minimal number of optical channels, thereby reducing system cost, computational load, and design complexity. By the end of the session, they hope to stimulate interdisciplinary dialogue and explore collaborations to co-develop the next generation of portable, non-invasive cerebral imaging tools that could transform neurocritical care and neonatal monitoring.

5.4 Confocal Microscopy: Different Setups Lead to Different Analysis of the Signals

Andreas Beutler (Mahr GmbH – Göttingen, DE, Andreas.Beutler@mahr.com)

The principle of confocal microscopy is presented. Included is the description of signal structure and challenges of signal analysis. A typical setup has been in practice for many years, however it has limitations for current requirements. We developed a new setup for a much faster and less complicated system which requires a different way of the analysis of the signals. First results are presented.

5.5 Feature Extraction with Applications in Watermark Recognition

Robert Beinert (Technical University of Berlin, DE, robert.beinert@tu-berlin.de)

Joint work of Robert Beinert, Matthias Beckmann, Jonas Bresch

The study of historical watermarks plays a major role in provenance research to determine the date and origin of paper-based writing and art. One of the main watermark collections is the so-called Wasserzeichen-Informationssystem (WZIS) that gathers watermarks from rubbings, handtracings, radiography, and thermography. Since the WZIS consists of nearly as many classes as samples, the training of common deep learning architectures does not yield reliable classifiers. Moreover, digitization techniques like the nowadays employed thermography are highly unstandardized, giving the reason to develop classifies that are invariant under affine image transformations. Based on the so-called Radon cumulative distribution transform (R-CDT), we therefore propose two easy-to-compute feature extractors that facilitate image classification tasks especially in the small data regime and guarantee linear separability of image classes that emerge from affine transformations. Studying the proposed max- and mean-normalized R-CDT, we show robustness against non-affine image deformations. Furthermore, the separability properties of both extractors are stable provided the Wasserstein distance between the samples can be controlled. Our theoretical results are supported by numerical experiments and may pave the path towards computational filigranology.

6 Exponential Analysis in Quadrature and Subdivision

6.1 Exponential Analysis Meets Quadrature and Subdivision

Annie Cuyt (University of Stirling, GB, annie.cuyt@stir.ac.uk)

Exponential analysis is a sparse reconstruction method of an exponential sum of the form

$$f(x) = \sum_{j=1}^{n} \alpha_j \exp(\phi_j x)$$

from $N \geq 2n$ of its equidistantly sampled values $f_s = f(s\Delta), \quad s = 0, \ldots, N-1$, where $\max_j |\Im(\phi_j)\Delta| < \pi$. While the ϕ_j can be obtained via the roots of the well-known formally orthogonal Prony polynomial

$$P_n(u) = \prod_{j=1}^{n} (u - \exp(\phi_j \Delta)),$$

the α_j are the solution of a Vandermonde structured linear system.

The nodes and weights of a Gaussian integration rule follow a similar scheme: the former are the zeroes of some orthogonal polynomial and the latter satisfy a Vandermonde linear system with the given moments f_s on the right hand side.

The annihilation of the values $\exp(\phi_j \Delta)$ by the Prony polynomial is also used in non-stationary subdivision schemes reproducing exponential polynomials, in particular to extract suitable ϕ_j from the data for the reproductive capability.

6.2 From Hermite to Zernike: Orthogonal Polynomials in Optics

Teresa E. Pérez (University of Granada, ES, tperez@ugr.es)

In 1865, Charles Hermite [1] published a paper (divided into four parts) introducing bivariate orthogonal polynomials on the disk to solve a bivariate approximation problem proposed by P. Chebyshev. Despite the fact that a priori the problem seems to be a simple generalization of standard orthogonal polynomials to the bivariate case, the solution presents several obstacles. C. Hermite then introduced the concept of biorthogonality in this context and orthogonal polynomials systems on the disk were described explicitly.

Zernike polynomials were introduced by Frits Zernike en 1934 [2] to describe the wavefront in the formation of images. In 2000, the Optical Society of America (OSA) adopted them as standard patron in Optics and Ophthalmology. Mathematically, Zernike polynomials are polynomials in two variables orthogonal on the unit disk, and are represented in polar coordinates as a product of a radial part (a univariate Jacobi polynomial) and an angular part represented by spherical harmonics.

In this talk we describe the families of bivariate orthogonal polynomials on the disk introduced by C. Hermite, show that Zernike polynomials are a particular case of disk polynomials, and we analyse the main applications of Zernike polynomial in Optics. Finally, our contributions in this topic are presented.

References

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6.3 Exponential Polynomial Reproduction in Subdivision: Annihilators and Symbol Factorization

Mariantonia Cotronei (Mediterranea University of Reggio Calabria, IT, mariantonia.cotronei@unirc.it)

Subdivision schemes are well-known iterative procedures for generating smooth curves or surfaces from discrete data. A typical requirement for such schemes is their ability to reproduce a function space, that is, to ensure that, starting from sampled data of a continuous function, exactly reconstruct that function in the limit. Focusing on spaces spanned by exponential polynomials, which are crucial in modelling curves of interest in CAGD, such as conics, spirals, or special trigonometric or hyperbolic functions, we explore the close relationship between subdivision schemes and Prony's problem. To do so, we first describe the kernel structure of both convolution and subdivision operators, emphasizing that exponential polynomial sequences are precisely all those that can be annihilated by such operators. We then show how, by making use of such property, the exponential polynomial reproduction capability of the scheme can be fully characterized by factorizing the (level-dependent) subdivision operator/symbol into specific factors that incorporate the exponential frequencies and their multiplicities. This reveals a strong analogy with Prony's method: while Prony's approach uses annihilating polynomials to recover exponential parameters from sampled data, the subdivision framework uses similar algebraic structures to encode such parameters into the design of the schemes.

6.4 Sobolev Orthogonal Polynomials and Spectral Methods in Boundary Value Problems on the Unit Ball

Miguel Piñar (University of Granada, ES, mpinar@ugr.es)

Our main objective in this work is to demonstrate how orthogonal Sobolev polynomials emerge as a useful tool within the framework of spectral methods for boundary-value problems. The solution of a boundary-value problem for a stationary Schrödinger equation on the unit ball can be studied from a variational perspective. In this variational formulation, a Sobolev inner product naturally arises. As test functions, we consider the linear space of polynomials satisfying the boundary conditions on the sphere, and a basis of mutually orthogonal polynomials with respect to the Sobolev inner product is provided. The basis of the proposed method is provided in terms of spherical harmonics and univariate Sobolev orthogonal polynomials. The connection formula between these orthogonal Sobolev polynomials and classical orthogonal polynomials on the ball is established. Consequently, the Sobolev Fourier coefficients of a function satisfying the boundary value problem are recursively derived. Finally, numerical experiments were presented.

7 Exponential Analysis in Engineering

7.1 Sparsity in Antenna Engineering

Dirk de Villiers (Stellenbosch University, ZA, ddv@sun.ac.za)

For the last several years a fruitful collaboration with the EXPOWER H2020 RISE project, and specifically Antwerp University, resulted in the application of various forms of sparse exponential analysis on antenna engineering problems. The talk presents several such example applications including:

- Direction of Arrival Estimation using 1-bit sampled data
- Near-field antenna position estimation using drone measurements
- Frequency ripple characterisation in reflector antenna noise temperate calculations
- Progress towards compact field pattern storage

7.2 Mathematical Challenges for Low-Frequency Radio Telescope Design

David B Davidson (ICRAR-Curtin, Perth, AU, David.Davidson@curtin.edu.au)

The overall theme of this paper is outstanding mathematical challenges in modelling (and designing) low-frequency (30-300 MHz) radio telescopes, epitomised by SKA-Low, the low-frequency component of the Square Kilometre Array radio telescope.

The presentation starts with a review of interferometric radio telescopes, and then moves on to introduce aperture arrays, which are receive-only phased arrays in conventional antenna terminology. Following this, Computational Electromagnetic (CEM) techniques [1] for simulating radio telescopes are addressed. The Method of Moments (MoM) has proven especially useful for wire antenna arrays, as widely used in low-frequency telescopes, and when combined with the Multi-Level Fast Multipole Method (MLFMM) acceleration technique, permits the analysis of large arrays, such as the 256-antenna SKA-Low "station" [2]. Nonetheless, these remain formidably large problems, with several million unknowns. Typical run-times for an SKA-Low station take hours to days for each frequency, depending on the rate of convergence of the MLFMM, even using high-performance computing resources.

Mutual coupling and its impact on telescope performance is a major theme of the talk. It is shown that the effect of mutual coupling is largely negative. The effects can be predicted but only by using a full simulation as above. In terms of SKA-Low science which may be impacted by this, the Epoch of Reionisation (EOR) is one of its major science cases. This requires looking for a signal five orders of magnitude in power below the foreground signals. The power spectrum approach is outlined, which uses Fourier analysis to attempt to distinguish the smooth foreground signal from the desired EOR signal. Recent work has demonstrated that mutual coupling poses major challenges to this approach.

The paper concludes with an outline of how new mathematical methods could assist in designing future low-frequency radio telescopes, combined with studies of more modest antenna systems with less overall complexity for initial work. Tools such as surrogate modelling could be very useful, but optimisation goals will need careful thought.

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7.3 Measuring Linear Array Mutual Coupling Terms using Exponential Analysis

Jacki Gilmore (Stellenbosch University, ZA, jackivdm@sun.ac.za)

We present a measurement-based method for jointly estimating the element positions and mutual-coupling coefficients of uniform linear arrays (ULAs) directly from embedded-element radiation patterns. The problem is cast as a Prony model whose shared exponential bases encode the electrical spacing (frequency and element locations), while the model coefficients capture the mutual coupling. By applying multi-snapshot validated exponential analysis (VEXPA) to the noisy pattern samples, the common bases are retrieved with sufficient resilience to noise. The coefficients are then obtained from an overdetermined Vandermonde system that is nearly perfectly conditioned. The technique is demonstrated on two cases:

- Synthetic example – An 11-element dipole ULA spanning 800–1200 MHz, sampled at 51 frequencies, with added white Gaussian noise (SNR = 60 dB). Using 361 pattern samples, the base terms and coefficients are both recovered. The relative error of the coefficients is in the order of 10-5,
- Measured array – A 4-element dipole ULA measured from 2.8–3.2 GHz at nine frequencies (again 361 samples). Both the base terms and the coefficients were extracted, allowing the mutual coupling terms to be determined with sufficient accuracy.

7.4 Selecting Sampling Rates and Sets for Efficient Super Resolution

Nuha Diab (Tel-Aviv University, IL, nuhadiab@tauex.tau.ac.il)

In the first part of the talk, we investigate the recovery of nodes and amplitudes from noisy frequency samples in spike train signals, also known as the super-resolution (SR) problem. When the node separation falls below the Rayleigh limit, the problem becomes ill-conditioned. Admissible sampling rates, or decimation parameters, improve the conditioning of the SR problem, enabling more accurate recovery. We propose an efficient preprocessing method to identify the optimal sampling rate, significantly enhancing the performance of SR techniques.

For the second part of the talk, we study the spectral properties of infinitely smooth multivariate kernel matrices when the nodes form a single cluster. We show that the geometry of the nodes plays an important role in the scaling of the eigenvalues of these kernel matrices. For the multivariate Dirichlet kernel matrix, we establish a criterion for the sampling set ensuring precise scaling of eigenvalues. Additionally, we identify specific sampling sets that satisfy this criterion. Finally, we discuss the implications of these results for the problem of super-resolution, i.e. stable recovery of sparse measures from bandlimited Fourier measurements.

7.5 Solving Estimation Problems Using Minimax Polynomials and Gröbner Bases

Akira Terui (University of Tsukuba, JP, terui@math.tsukuba.ac.jp)

An estimation problem is a problem in which one infers or estimates a certain quantity or state based on uncertain information or observed data. Estimation problems play a crucial role across various disciplines, including statistics, machine learning, signal processing, and control theory. To solve estimation problems, numerical methods such as the gradient method or genetic algorithms are used. However, gradient methods may return a local solution, depending on the initial values, since they utilize local convergence properties. The genetic algorithm has some disadvantages, as it sometimes fails to properly solve the estimation problem due to phenomena such as initial convergence and hitchhiking. On the other hand, using minimax approximation together with Gröbner bases computation may avoid these phenomena, for this method evaluates values globally. In this presentation, we propose a method for solving estimation problems using minimax polynomials and Gröbner bases. We show an application of the proposed method for solving a speech direction estimation problem.

8 Computer Algebra

8.1 What's New in Maple 2025

Jürgen Gerhard (Maplesoft – Waterloo, CA, jgerhard@maplesoft.com)

We will highlight some of the new features in Maple 2025, including for advanced mathematical computations, user interface redesign, graph theory, visual expression comparison, code generation, and programming.

8.2 Sparse Interpolation in Chebyshev Basis: Early Termination and Georg Heinig's Toeplitz Solver

Erich Kaltofen (North Carolina State University – Raleigh, US, kaltofen@ncsu.edu)

Joint work of Erich Kaltofen, Zhi-Hong Yang

Ideas by Kaltofen and Yang [1] for error-correcting interpolation of polynomials that are a sparse linear combination of Chebyshev polynomials have led to a new early termination algorithm for computing the sparsity.

Kaltofen and Lee [2] in their early termination algorithms used thresholds to skip over sporadic probabilistic errors. For early termination in sparse Chebyshev interpolation, thresholds need an algorithm to step from a sequence of singular leading principal submatrices of a Toeplitz matrix to the next non-singular leading principal submatrix. For Prony sparse interpolation, the problem is solved by the 1969 Berlekamp-Massey algorithm, and for Chebyshev sparse interpolation by Georg Heinig's 1983 Toeplitz algorithm.

In my talk, I will describe our new early termination algorithm and Heinig's Toeplitz solver from a Berlekamp-Massey algorithmic viewpoint. Heinig's algorithm, which generalizes the classical Toeplitz solvers by Levinson and Durbin, takes quadratic time and requires linear space.

This is joint work with Zhi-Hong Yang at Central South University, China.

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8.3 Quasi-Linear Interpolation of Sparse Polynomials Over the Integers

Bruno Grenet (LIRMM, University of Montpellier, CNRS Montpellier, FR, bruno.grenet@lirmm.fr)

Joint work of Bruno Grenet, Pascal Giorgi, Armelle Perret du Cray, Daniel S. Roche

Prony's method can be used for sparse interpolation over any ring: Given black box access to a polynomial with t non-zero terms, its coefficients and exponents can be computed from evaluations on a geometric sequence of size $2t$. However, over exact rings such as the integers or finite fields, this algorithm is computationally expensive. Several techniques have been developed by the computer algebra community to speed up the algorithm. As a result, a sparse polynomial over the integers can be interpolated at a cost that is quasi-linear in the size of its sparse representation.

8.4 Fast Interpolation and Multiplication of Unbalanced Polynomials

Pascal Giorgi (LIRMM, University of Montpellier, CNRS Montpellier, FR, pascal.giorgi@lirmm.fr)

Joint work of Pascal Giorgi, Bruno Grenet, Armelle Perret du Cray, Daniel S. Roche

Efficient polynomial or integer multiplication is at the core of computer algebra and these problems received a lot of attention since the last past decades to reach quasi-linear time complexity. Nowadays, it is even possible to multiply polynomials with integer coefficients within a quasi-optimal complexity. Note that the latter result assumes that the bit-lengths of the coefficients must not vary too much, meaning that polynomials with very unbalanced coefficients are out of reach yet. In this talk, I will show how this problem of unbalanced integer polynomials multiplication is related to sparse polynomial interpolation. By using our recent technique on sparse interpolation for integer polynomial, we show how we can reach an almost quasi-optimal complexity for the multiplication problem. In particular, we will describe a new algorithm that enables the interpolation of sparse unbalanced polynomials in almost quasi-linear time.

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