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Algorithms and Complexity for Continuous Problems

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Dagstuhl-Seminar

Algorithms and Complexity for Continuous Problems

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April 15–19, 1991

Contents

This Seminar-Report contains an overview and the abstracts of 30 lectures in alphabetical order. One of these lectures could not be presented on the Seminar.

We also include a summary of two plenary sessions on "New Research Directions, Open Questions, Discussion" but of course we can not summarize the many informal talks which were stimulated by the great atmosphere of Schloß Dagstuhl.

Overview

Many problems in science and engineering are described by continuous models. Yet, most work to date on computational complexity has been for combinatorial or algebraic complexity. Recently there has been considerable research on the complexity of continuous problems and the Dagstuhl-Seminar was devoted to this subject.
The topics, listed below in alphabetical order, are typical of the subjects covered at the Workshop. We list both theoretical and applied topics.

Theoretical Topics

Average Case Complexity
Complexity of Nonlinear Problems
Complexity of Parallel Computation
Computational Models
Intractable Problems
Noisy Data
Probabilistic Complexity
Randomization (Monte Carlo Methods)
Worst Case Complexity

Applications

Approximation
Control Theory
Ill-Posed Problems
Integral Equations
Large Eigenvalue Problems
Large Linear Systems
Linear Optimization
Multivariate Integration
Nonlinear Equations
Nonlinear Optimization
Ordinary Differential Equations
Partial Differential Equations
Prediction and Estimation
Signal Processing
Zeros of Systems of Polynomials

Publications

Research papers which fall within the scope of this Workshop appear in many journals. The *Journal of Complexity* is primarily devoted to the complexity of continuous problems. Many papers which were presented on this Dagstuhl-Seminar will be published in this Journal.

Abstracts

The Average-Case Complexity of the Simplex-Method

Karl Heinz Borgwardt
Universität Augsburg

Ten years ago the author had shown that the average number of pivot steps required for solving linear programming problems by the Simplex method is polynomial when we consider the following stochastic model:
problem: $\max v^T x$ such that $a_1^T x \leq b^1, \ldots, a_m^T x \leq b^m$ $(b^i > 0)$
distribution: $v, a_1, \ldots, a_m$ distributed symmetrically under rotations, identically and independently on $\mathbb{R}^n$.
All the problems generated under this model were feasible and a feasible point (0) is immediately at hand. The analysis and the polynomiality proof had been based on the availability of a feasible point. Still the question was open, whether in a corresponding model with arbitrary b^i the problem of finding a feasible point is significantly harder than the final optimization.
In this talk the author answers this question and proves that the process of finding a feasible point is of the same complexity-order as the optimization-phase. The tool for that result is another application of the shadow-vertex-algorithm with a homogenization step. It is demonstrated, how the average number of shadow-vertices can be calculated, when the restriction vectors follow a cylindrical distribution, i.e., rotation-symmetric in n dimensions and uniform in dimension $n+1$.

Sparsity and Nonlinearity

David Donoho
Stanford University and University of California, Berkeley

It is known that for certain sets $\mathfrak{F}$, the approximation problem of recovering $f(x)$, $x \in [0,1]$ from $y_i = L_i(f)$, $i = 1, \ldots, n$, L_i linear, admits cases where nonlinear methods $\widehat{f} = \Phi(L_1(f), \ldots, L_n(f))$ are essentially better then every linear method, with respect to $\inf_\Phi \sup_{f \in \mathfrak{F}} \|\widehat{f} - f\|_{L_2[0,1]}$. For example if $\mathfrak{F} = W^{m,p}$, $p < 2$, then nonlinear methods are better. A similar phenomenon occurs in the case $y_i = f(t_i) + z_i$, $i = 1, \ldots, n$, $z_i \overset{\text{iid}}{\sim} N(0, \sigma^2)$, $f \in \mathfrak{F}$, with respect to $\inf_\Phi \sup_{f \in \mathfrak{F}} E\|\widehat{f} - f\|^2_{L_2[0,1]}$. Again this happens if $p < 2$ (Nemirowskii). In fact there is a connection between the two phenomena.
In seeking to find a documentation of the phenomenon which would be plausibel to scientists working with real problems, we studied a variety of scientific areas where, empirically, sparsity is present and nonlinear methods perform much better than

linear ones. Here sparsity means that the object to be recovered is usually zero in most coefficients.
We need this insight to show that the phenomenon of nonlinear algorithms outperforming linear ones was present in $W^{m,p}$. We discovered that the same phenomenon of sparsity was also present, which suggests new and very simple algorithms for smoothing noisy data by shrinking the empirical wavelet coefficients.
This is joint work with Iain Johnstone.

Optimal Quadrature and the Least Norm of Monosplines with Free Knots on the Real Axis

Fang Gen Sun
Beijing Normal University
(joint work with Lin Yong Pin)

We study the problem of optimal quadrature for the class $W_1^r(\mathbb{R}) = \{f \in L_1(\mathbb{R}) \mid f^{(r-1)}$ is abs. cont. on every finite interval and $\|f^{(r)}\|_1 \leq 1\}$ with Hermite Birkhoff type of information for the whole real axis, and under some conditions we obtain the optimal quadrature formula and the optimal error.
We also study the exact solution of the minimization of the L_∞-norm on some classes of monosplines with free knots on the real axis. The optimal quadrature formula and the optimal error on the convolution class taking a Polya frequency density as its kernel are given.
The first author is visiting the Mathematical Institute of the University Erlangen-Nürnberg.

Barnsley's Scheme for the Fractal Encoding of Images

Siegfried Graf
Universität Passau

An iterated function system (IFS) is an N-tupel of contractions $w_1, \ldots, w_N : \mathbb{R}^m \to \mathbb{R}^m$. By a result of Hutchinson each IFS has a unique attractor A, i.e., A is the unique nonempty compact set with $A = w_1(A) \cup \ldots \cup w_N(A)$. It can be shown that every non-empty compact set K can be approximated to any given accuracy (w.r.t. the Hausdorff metric h) by the attractor of an IFS consisting of affine maps. It was Barnsley's idea to encode an image up to a given accuracy by the affine maps generating an approximating attractor. A heuristic method to obtain such an encoding is provided by Barnsley's collage theorem: If $(w_1, \ldots, w_N)$ is an IFS with attractor A and if $\emptyset \neq L \subset \mathbb{R}^m$ is compact with $h(L, w_1(L) \cup \ldots \cup w_N(L)) < \varepsilon$ then $h(L, A) < \varepsilon/(1-s)$ where s is the maximum of the contraction constants of $w_1, \ldots, w_N$. The (approximate) reproduction of the image can be done by means of Barnsley's random iteration algorithm.

Complexity in Symbolic Solving of Differential Equations

D. Grigoriev
Steklov Institute, Leningrad Department

This is a survey on the different results in symbolic solving of differential equations in different classes of explicitely given functions. Namely, the formal solvability is considered, solvability in computable, analytic functions, in formal power series. Also the problems of manipulating in the ring of ordinary linear differential operators are considered, in particular, calculating the greatest common divisor of a family of operators, factoring of an operator, and solving linear systems over this ring. Complexity questions in the area under discussion are considered and investigated for the first time.

Complexity of Monte Carlo Solution of Integral Equations

Stefan Heinrich
Karl-Weierstraß-Institut für Mathematik, Berlin

There is a permanent discussion about the advantages or disadvantages of Monte Carlo methods in numerical analysis. While most of these arguments are heuristical or experimental, complexity theory for continuous problems created the tools to put such issues onto a firm mathematical basis. In the earlier sixtieth Bakhvalov studied the efficiency of stochastic integration. Later on, approximation and optimization followed. The complexity of Monte Carlo solution of integral equations remained open.

This talk presents the solution to this problem. The classical von Neumann-Ulam scheme is far from the optimal order. It turns out that the optimal rate is obtained by a deterministic preconditioning process, followed by the classical Monte Carlo algorithm applied to these preconditioned data. The optimal rate is the product of the classical Monte Carlo rate and the optimal deterministic rate, so it unifies the advantages of both approaches. Numerical perspectives of this procedure are discussed, as well.

Complexity of Linear Problems with Nonexact Information

Bolesław Z. Kacewicz
University of Warsaw
(joint work with L. Plaskota)

We deal with approximating linear operators based on information corrupted by a noise. Information consists of nonexact values of linear functionals. Pertubations in information can be absolute or relative, and are assumed to be bounded, each

bound dependent on a consecutive number of a functional. For a class of problems we find the minimal information cost $MC(\varepsilon)$ of obtaining an ε-approximation in the worst case setting, for a given positive accuracy ε. We determine the optimal (up to a constant) number of functionals, optimal precisions with which they should be obtained, and optimal information and algorithm. We next discuss the information cost in the asymptotic setting. It turns out that for any nonexact information and algorithm the 'difficult' problems, for which the cost of obtaining an ε-approximation is at least $MC(\varepsilon)$, form a dense set in the problem element space. On the other hand, the minimal cost $MC(\varepsilon)$ is achieved for some information and algorithm.
The results are applied to the problem of recovering functions in s variables with r continuous derivates with information given by finite precision representation of function values. The minimal cost, measured by the number of bits sufficient to store the information from which an ε-approximation can be obtained, is proportional to $\varepsilon^{-s/r} \log_2(1/\varepsilon)$.

Parallel Complexity of Numerical Problems

Jörg-Detlef Kern,
Karl-Weierstraß-Institut für Mathematik, Berlin
(joint work with S. Heinrich)

We develop the theory of information-based complexity from a parallel point of view. We use a model of computation with p processors, each being capable of arithmetic operations and decisions, and analyze the full numerical process starting from obtaining informations up to the arithmetic computation of the approximate values. This is done for three basic numerical problems: function approximation, integration, and solution of Fredholm integral equations of the second kind. In each case the complexity is studied as a function of three parameters - the required precision ε, the number p of processors, and the number of values to be computed. For all problems we are able to determine the order of the complexity as a function of all three variables simultaneously.
Major ingredients of our investigations are the results of Ben-Or on the complexity of algebraic decision problems and a variant of parallel binary search.

The Worst Case Limit of Probabilistic Complexity

Mark Kon
Boston University

We compare worst case and probabilistic complexity in the information-based complexity setting. We show that for a linear map ("problem") S between two Banach spaces F and G, the following are true:

(a) If F and G are Banach, and G is uniformly convex, then the $\delta = 0$ case of probabilistic complexity is identical to worst case complexity. That is, if $\mathrm{card}(\varepsilon, \delta)$ denotes the number of information operation necessary to get an accuracy ε with probability $1 - \delta$, then $\mathrm{card}(\varepsilon, 0) = \mathrm{card}(\varepsilon)$, where the later denotes the worst case complexity. Essentially all properties in the $\delta = 0$ case are the same as worst case properties.
(b) If F and G are Hilbert spaces, then $\lim_{\delta \to 0} \mathrm{card}(\varepsilon, \delta) = \mathrm{card}(\varepsilon)$, i.e., the $\delta \to 0$ limit of probabilistic is identical to worst case complexity.

Encoding and Recovery of Operator Values

Nikolaj P. Korneichuk
Ukrainian Academy of Sciences, Kiev

Let X and Y be metric spaces, and let A be a continuous operator from X into Y. If $M_N = (\mu_1, \ldots, \mu_N)$ is a collection of continuous functionals on X, then to each element $x \in X$ we assign a numerical vector $T(x, M_N) = (\mu_1(x), \ldots, \mu_N(x))$. The amount of information of the encoding method M_N is measured through the diameter of the set $\{y \in Y \mid y = Az,\ z \in X,\ T(z, M_N) = T(x, M_N)\}$. For some sets $\mathfrak{M} \subset X$ we consider the problem of exact estimation of this diameter. We also study the effective recovery of elements $y = Ax$ by the information $T(x, M_N)$ and the optimization of the encoding method. Concrete situations are considered when A is the operator of m-th differentiation or a convolution operator, and also when A is the operator of solution of a boundary-value problem.

Recovering Band-limited Signals from Inaccurate Data

Marek Kowalski
University of Warsaw
(joint work with Bolesław Kacewicz)

The current presentation concerns recovering linear continuous functionals on unitary spaces with applications to reconstructing band-limited signals from inaccurate information. The motivation comes from real-life computations in which available data is usually contaminated, e.g., by observational or rounding errors. We show how data perturbations alter the minimal recovery error and specify the form of an optimal algorithm.
We consider the problem of recovering a signal $\check{x}$ from its nonexact samples $\check{x}(t_1)$, $\ldots, \check{x}(t_n)$ and show that if δ is the accuracy of data reading (measured in L_2 norm) then the recovery error essentially depends on $\delta/\sqrt{\lambda_{\min}}$. Here $\lambda_{\min}$ is the minimal eigenvalue of the matrix $\mathfrak{M} = (\sin(\Omega(t_j - t_k))/(\Omega(t_j - t_k)))_{j,k=1}^n$ with Ω beeing the

signal band level. This suggests that sampling points should be chosen to maximize $\lambda_{\min}$.
We also ponder the recovery from noisy values $(\check{x}, \phi_0), (\check{x}, \phi_1), \ldots, (\check{x}, \phi_{n-1})$, where $(\cdot, \cdot)$ is the inner product in $L_2[-\tau, \tau]$ and ϕ_j are suitably normalized prolate spheroidal wave functions on the interval $[-\tau, \tau]$. We show that perturbations in the data additively affect the recovery error by δ.

Estimating the Smallest Eigenvalue and Condition Number of a Symmetric Positive Definite Matrix by the Lanczos Algorithm with a Random Start

Jacek Kuczynski
Polish Academy of Sciences, Warsaw
(joint work with Henryk Woźniakowski)

The Lanczos algorithm with a randomly chosen starting vector for estimating the smallest eigenvalue of a symmetric positive definite matrix A is analyzed. We obtain upper bounds on the normalized Lebesgue measure (probability) of the set of those starting vectors for which the Lanczos algorithm fails to give an ε-approximation at the k-th step. We also provide upper bounds on the expected relative error of the Lanczos algorithm with respect to all starting vectors from the unit sphere. Combining these results with the corresponding theorems for approximating the largest eigenvalue we obtain some estimates on the condition number of A in the 2-norm.

A Minimax Principle in Information-Based Complexity

Peter Mathé
Karl-Weierstraß-Institut für Mathematik, Berlin

Minimax theorems are well known and often applied in statistics and game theory. The talk gives a result of this type of theorems under asumptions usually met in information-based complexity. As a consequence we obtain, that the optimal approximation rate of (continuous) Monte Carlo methods coincides with the worst (w.r.t. the measure on the class of problem elements) performance in the average case setting. The proof gives additional insight on the nature of the "least favorable" distributions and on the optimal Monte Carlo methods.
In case of allowing only linear deterministic methods by realization of our Monte Carlo process additional information is given. The "least favorable" distribution is sitting at the extreme points of our set of problem elements. The machinery developed allows to compute some optimal Monte Carlo approximation rates exactly. The case of a diagonal mapping acting between l_1 and l_2 is studied to show that.

Real Number Models and the Use of Information

Klaus Meer
Technische Hochschule Aachen

The "$P = NP$"-question in classical complexity theory was characterized by Krentel via the use of information. He showed the equivalence of $P = NP$ and $P^{NP(p(n))} = P^{NP(O(\log n))}$, where $P^{NP(t(n))}$ is the class of deterministic polynomial-time machines using an NP-oracle at most $t(n)$ times during a computation on an input of size n (p some polynomial). In the proof, given an NP-machine M and an input X, a polynomially sized set $A(X)$ of paths is constructed such that X is accepted by M iff there exists an accepting path in $A(X)$. Crucial for this approach is the fact that for any given path there exists a compatible guess.

This situation changes when using real number models like that of Blum, Shub, and Smale. Here the problem: "given a computation path of an $NP_{\mathbb{R}}$-machine, does there exist a compatible guess" is already $NP_{\mathbb{R}}$-complete. Nevertheless Krentel's theorem can be transformed to the decision problem $(F^2, F^2_{\text{zero},+})$. This is the problem of deciding whether an arbitrary polynomial $f : \mathbb{R}^n \to \mathbb{R}$ of degree two has a nonnegative zero ($n \in \mathbb{N}$). This can be done by using sets of vectors instead of sets of paths: assume that the above condition concerning the use of information is valid for BSS-machines. Then for a given degree-two polynomial f ($f(0) > 0$, without loss of generality) a polynomially sized set $B(f)$ of nonnegative vectors can be obtained such that $f \in F^2_{\text{zero},+}$ iff there exists a vector $x \in B(f)$ with $f(x) \leq 0$. This leads straightforward to a polynomial-time algorithm for $(F^2, F^2_{\text{zero},+})$. Hence $P_{\mathbb{R}}^{NP(p(n))} = P_{\mathbb{R}}^{NP(O(\log n))}$ implies $(F^2, F^2_{\text{zero},+}) \in P_{\mathbb{R}}$.

It is an open question whether Krentel's theorem also holds for the $NP_{\mathbb{R}}$-complete problem (F^4, F^4_{zero}). Here $F^4 = \{f : \mathbb{R}^n \to \mathbb{R} \text{ degree-four polynomial}, n \in \mathbb{N}\}$ and $F^4_{\text{zero}} = \{f \in F^4 \mid \exists x \in \mathbb{R}^n : f(x) = 0\}$.

Application of Information-Based Complexity to Computational Transport Theory

Paul Nelson
Texas A&M University, College Station

This lecture is motivated by the desire to apply information-based complexity to compare known methods for solving neutron transport problems. Attention is restricted to methods that are discrete counterparts of the classical source iteration procedure. A one-dimensional problem modelling the interior sweeps of this iteration is introduced. For two classes of information, cell-average and point-evaluation (at all centers) we compute radii of informations and determine optimal algorithms, on various problem domains. We also study the classical diamond-difference and

step-characteristic methods, the former for both types of information and the later for cell-average information only. For the later information it is shown that the diamond-difference method has the smaller worst-case error. For cell-average information in two spatial dimensions we give an optimal algorithm. For the further case of a single square cell we compare the worst-case errors of three known methods (step-characteristic, diamond-difference, and (c, c)-model).

Analytical Complexity of Iterative Solution of Linear Operator Equations

Arkadii Nemirowskii
Academy of Sciences of the USSR, Moscow

The talk presents results on the complexity of the equations $Ax = b$ (A is a bounded linear operator on a Hilbert space H, $b \in H$) with respect to iterative "matrix-vector-multiplication-based" procedures, i.e., the procedures based on the oracle which can multiply a given $h \in H$ by A and the conjugate operator A^*. For a family of classes of equations and accuracy measures sharp complexity bounds are given; suboptimality of the conjugate gradient method is established. We also consider the case of incorrect data and present extensions of the complexity results onto smooth convex minimization problems.

Efficient Methods for Multidimensional Numerical Integration

Harald Niederreiter
Austrian Academy of Sciences, Vienna

After a brief background on quasi-Monte Carlo integration, the theory of lattice rules for multidimensional numerical integration is discussed in detail. We first periodize the integrand such that it is periodic with period interval $[0,1]^s$. For such integrands we consider the regularity class $\mathfrak{E}_k^s$, $k > 1$, characterized by the property $\widehat{f}(\underline{h}) = O(r(\underline{h})^{-k})$, where $r(\underline{h}) = \prod_{i=1}^s \max(1, |h_i|)$ for $\underline{h} = (h_1, \ldots, h_s) \in \mathbb{Z}^s$. A lattice rule L uses the approximation

$$\int_{[0,1]^s} f(\underline{x})\, d\underline{x} = \frac{1}{N} \sum_{n=1}^{N} f(\underline{x}_n),$$

where $\underline{x}_1, \ldots, \underline{x}_N \in [0,1)^s$ are such that the residue classes $\underline{x}_1 + \mathbb{Z}^s, \ldots, \underline{x}_N + \mathbb{Z}^s$ form a subgroup of $\mathbb{R}^s/\mathbb{Z}^s$. For $f \in \mathfrak{E}_k^s$ the integration error is $O(R_k(L))$, where $R_k(L) = \sum r(\underline{h})^{-k}$ with the sum extended over all $\underline{h} \neq \underline{0}$ in the dual lattice $L^\perp$ of L. Various classical methods lead to results which guarantee the existence of N-point lattice rules with $R_k(L) = O(N^{-k}(\log N)^{ks})$. Very recent results of the speaker demonstrate

the existence of N-point lattice rules with $R_k(L) = O(N^{-k}(\log N)^{k(s-1)+1+\varepsilon})$, and even with $R_k(L) = O(N^{-k}(\log N)^{k(s-1)})$ in the case the number $\tau(N)$ of positive divisors of N satisfies $\tau(N) = O((\log N)^{s-1})$ (which is true for "most" N since the average order of magnitude of $\tau(N)$ is $\log(N)$). For lattice rules of rank 1 it is known that $R_k(L)$ is at least of the order of magnitude $N^{-k}(\log N)^{s-1}$.

Quadrature Formulas for Monotone Functions

Erich Novak
Universität Erlangen-Nürnberg

We prove that adaptive quadrature formulas for the class of monotone functions are much better than nonadaptive ones, if the average error (Dubins-Freedman-Ulam measure) is considered. Up to now it was only known that adaptive methods are not better in the worst case (for this and many other classes of functions, Kiefer (1957) and Bakhvalov (1971)) or in various average case settings (e.g. Wasilkowski & Woźniakowski (1984)).
We also prove that adaptive Monte Carlo methods are much better than nonadaptive ones. This also contrasts with analogous results for other classes (Sobolev classes, Hölder classes) where adaption does not help much.
Though adaptive Monte Carlo methods are studied in the literature with respect to lower bounds, we construct for the first time an adaptive method to get better upper bounds. Our new Monte Carlo method can easily be implemented.

Information Complexity of Equations of the Second Kind with Compact Operators in Hilbert Space

Sergej V. Pereverzev
Ukrainian Academy of Sciences, Kiev

Let $\{e_i\}_{i=1}^{\infty}$ be the orthonormal basis of a Hilbert space X and let P_n be the orthogonal projector on $\operatorname{span}\{e_i\}_{i=1}^{n}$. We denote by X^ν, $0 < \nu < \infty$, the normed subspace of X which satisfies the conditions $\|\varphi - P_n\varphi\|_X \le cn^{-\nu}\|\varphi\|_{X^\nu}$ and $\|\varphi\|_X \le \|\varphi\|_{X^\nu}$ for any $\varphi \in X^\nu$ and $n \in \mathbb{N}$. Let $\mathfrak{L}(X, X^\nu)$ be the space of all linear and continuous operators H acting from X to X^ν with the usual norm; and let X_0^ν be the unit ball of the subspace X^ν.
For some $\mathfrak{H} \subset \mathfrak{L}(X, X^\nu)$ we denote by $\Psi_{\mathfrak{H}}^\nu$ the class of the equations of the second kind

$$z = Hz + f$$

where $H \in \mathfrak{H}$ and $f \in X_0^\nu$. To each equation of this form we assign the numerical vector $T_m(H, f) = ((e_i, He_j), (f, e_k))_{(i,j)\in\Gamma_m, k=1,2,\ldots,2^m}$, where Γ_m is the symmetric

hull of the plane set

$$\{1\} \times [1, 2^{2m}] \cup \bigcup_{k=1}^{m} (2^{k-1}, 2^k] \times [1, 2^{2m-k}].$$

Now we introduce the next class of operators

$$\begin{aligned} \mathfrak{H}^{\nu,\mu} &= \mathfrak{H}^{\nu,\mu}(\alpha, \beta, \gamma) \\ &= \{H \in \mathfrak{L}(X, X^\nu) \mid \|H\|_{X \to X^\nu} \le \alpha,\ \|(I - H)^{-1}\|_{X \to X} \le \beta, \\ &\qquad H^* \in \mathfrak{L}(X, X^\mu),\ \|H^*\|_{X \to X^\mu} \le \gamma\}. \end{aligned}$$

The following theorem is the main result of this report. If for K. I. Babenko's perturbated width $\delta_N(X_0^\nu, X)$ we have the estimate $\delta_N(X_0^\nu, X) \ge c\, N^{-\nu}$ for $N \in \mathbb{N}$, then for any $\mathfrak{H} \subset \mathfrak{H}^{\nu,\mu}$ and $\mu/2 \le \nu \le \mu$ the estimate

$$c\,\varepsilon^{-1/\nu} \le \mathrm{comp}(\Psi_{\mathfrak{H}}^\nu, \varepsilon) \le c_1 \varepsilon^{-1/\nu} \log^{1+1/\nu} 1/\varepsilon$$

holds for the ε-complexity in the sense of J. Traub and H. Woźniakowski. The information $T_m(H, f)$, $m\, 2^{2m} \asymp (1/\varepsilon)^\nu$ is order-optimal in the power scale in the sense of the ε-cardinality. Moreover, we construct some special algorithm, which is order-optimal in the sense of ε-complexity.
The theorem will allow to obtain the exact order of ε-complexity for classes of Fredholm equations, Volterra equations, and weakly singular integral equations.

Approximation and Integration Problems on the Wiener Space with Noisy Data

Leszek Plaskota
University of Warsaw

We study function approximation and integration problems on the Wiener space of functions $F = \{f \in C^{(r)}([0,1]) \mid f(0) = \ldots = f^{(r)}(0) = 0\}$, in the framework of the average case setting of information-based complexity. We assume that available information is always contaminated by some Gaussian noise and consider standard information consisting of function values or its derivatives. We show that the n-th optimal error for function approximation behaves as $(\sigma^2/n)^{1/4} + 1/\sqrt{n}$ for $r = 0$, and as $\sigma/\sqrt{n} + (1/n)^{r+1/2}$ for $r \ge 1$ (σ is a global parameter corresponding to the noise level). For integration the n-th minimal error is $\sigma/\sqrt{n} + (1/n)^{r+1}$. Optimal information is given by function values at equidistant points. These results lead to a rather surprising conclusion that for the approximation problem with $r = 0$ the information consisting of arbitrary linear and continuous functionals is much more powerful than standard information, since the first gives the error $\sigma \ln n/\sqrt{n}$. Optimal algorithms are special smoothing splines.

Average Error Bounds for Global Optimization and Zero Finding

Klaus Ritter
Universität Erlangen-Nürnberg

We study the global optimization problem on the class of continuous functions $f : [0,1] \to \mathbb{R}$ with $f(0) = 0$. The location of the global maximum of f is estimated by a nonadaptive method S_n using function values at n fixed nodes. The quality of S_n is defined as the average of $\max f - f(S_n(f))$ with respect to the Wiener measure. It turns out that equidistant nodes are asymptotically optimal (but not optimal in general) and yield a rate $n^{-1/2}$ of the error. Clearly it is not sufficient to consider only nonadaptive methods, since adaption helps on the average for this global optimization problem. However, we are not able to quantify the improvement which is due to adaption.

Another nonlinear problem is the estimation of a zero of a continuous function $f : [0,1] \to \mathbb{R}$ with $f(0) = -1$ and $f(1) = 1$. Here we consider adaptive (i.e., sequential) methods S_n using n function values, and the quality of S_n is given by the average of $d(f^{-1}(0), S_n(f))$ (root criterion) with respect to the Brownian bridge. Upper error bounds of the form α^n, $\alpha < 1/2$, are known for this problem, and we prove a lower bound β^n with $\beta > 0$. Hence, on the average one can only achieve linear convergence of the error (joint work with Erich Novak).

Iterative Methods and the R-Order of convergence

Jochen W. Schmidt
Technische Universität Dresden

The rate of convergence is an important measure to charcterize the local behaviour of iterative processes. Several concepts of convergence order have been introduced. The most essential nowaday are the Q- and the weaker R-order. In general in an iterative process several sequences $(x_{n,1}), \ldots, (x_{n,s})$ are computed simultaneously or sequentially. In the lecture it was assumed that a system of inequalities

$$0 \le \varepsilon_{n+1,i} \le \alpha_i \prod_{j=1}^{s} \varepsilon_{n,j}^{m_{i,j}} \prod_{j=1}^{i-1} \varepsilon_{n+1,j}^{r_{i,j}} , \qquad i = 1\,(1)\,s$$

is given by which the error terms of the current iteration step are estimated by those of the previous step. The results presented in the lecture lead to a straightforward procedure for computing the best R-orders of all sequences in question by using the information $m_{i,j}$ and $r_{i,j}$. With a number $\tau > 1$ and a vector $\mu = (\mu_1, \ldots, \mu_s)^T > 0$ assume that $M\mu + \tau R\mu \ge \tau\mu$, where $M = (m_{i,j})$ and $R = (r_{i,j})$. Then, for sufficiently small initial values all sequences converge at least with the R-order τ. Moreover, the arising optimization problem "maximize τ such that $M\mu + \tau R\mu \ge \tau\mu$, $\mu \ge 0$, $\mu \ne 0$" has the optimal value $\tau_{\max} = \rho((I - R)^{-1}M)$ where ρ denotes the

spectral radius. Further, if $\tau_{\max} > 1$, there are sequences satisfying the above system of inequalities whose R-orders are arbitrarily close to $\tau_{\max}$, i.e., the value $\rho((I - R)^{-1}M)$ is the best R-order that can be derived from this system of inequalities. Ref.: W. Burmeister/J. W. Schmidt: Numer. Math. **53** (1988).

Optimal and Universal Multidimensional Quadrature Formulas

Volodya N. Temlyakov
Steklov Institute, Moscow

The main results of this report are devoted to constructing good multidimensional quadrature formulas (cubature formulas) for classes of functions with bounded mixed derivative. It turns out that these questions are closely connected with deep problems in number theory. For constructing concrete good cubature formulas different methods are used, beginning with sufficiently elementary methods for investigating Fibonacci and Korobov cubature formulas and finishing with cubature formulas based on algebraic properties of real roots of irreducible polynomials.
We discuss in this report the fact that Fibonacci cubature formulas ($d = 2$) and cubature formulas $\phi(a, A, n)$ constructed by using real roots of irreducible algebraic polynomials of degree d ($d \geq 3$) are as optimal for classes of functions with bounded mixed derivative as for Sobolev classes with an arbitrary anisotropy. This is a very important property of universality of these cubature formulas.

Robustness: The Computational Complexity Point of View

Roberto Tempo
CENS-CNR Politecnico di Torino

In this talk we present some preliminary results on the computational complexity of the robust stability problem. That is, we evaluate the number of elementary operations needed to check if all roots of an n-th order interval polynomial $p(s, q)$ are contained in a given region $\mathfrak{D}$ of the complex plane.
First, we study the case when $\mathfrak{D}$ is the open left half plane and show that the number of operations is $O(n^2)$. This number is obtained combining Kharitonov's Theorem and Routh's algorithm. As a second example, we take $\mathfrak{D}$ equal to the unit disc and consider a class of interval polynomials having perturbations on about half the coefficients. Even in this special case the number of elementary operations is $O(2^n - n^2)$ and is obtained combining the extreme point results of Hollot and Bertlett and the test of Jury. As a final negative example, we study the case when $\mathfrak{D}$ is a "spacial sector" contained in the left half plane. Unfortunately a combination of the "strong" extreme point result of Soh and Foo and the eigenvalue test of Davison and Remerk gives only an iterative solution to the problem.

Average Dimension and γ-Widths of Classes of Functions on the Whole Line

V. M. Tikhomirov
Moscow State University
(joint work with G. G. Magaril-Il'yaev)

From the introduction. The first works devoted to the problem of approximation of functions on $\mathbb{R}$ were written by Sergej Bernstein in the thirties. He considered the space of entire functions of exponential type as a tool of approximation. But during the last decades splines are more and more often used as an approximating set. Spaces of entire functions and splines are of infinite dimension and quantities, characterizing the corresponding approximation, are expressed in terms reflecting the inner structure of the approximating set (such as the order of the entire function or the density of the distribution of spline knots). The following question arises: How can we compare these methods of approximation?

We present an approach to this problem which is connected with the concept of average dimension. Shannon was the first who considered the problem of average characteristics, connected with the notion of average entropy. Kolmogorov introduced a non-random version of this concept and the first result in that direction was obtained by Tikhomirov (1957). An analogous characteristic, namely average dimension, which is based on the notion of Kolmogorov's n-widths, was proposed by Tikhomirov (1980) and the program of exploration was outlined.

The program connected with the calculation of various average widths and the description of optimal subspaces and operators has been realized by G. G. Magaril-Il'yaev during the last three years. Nowadays similar problems are also considered in Beijing Normal University by Sun Yongsheng and his students. In this paper we give the main results obtained by G. G. Magaril-Il'yaev.

(This invited lecture could not be presented on the Seminar.)

Some Thoughts on the Nature of Continuous Computational Complexity

Joseph F. Traub
Columbia University, New York

A typical paradigm of science is shown in Figure 1. A mathematical formulation of a natural phenomenon is created. Computations stemming from the mathematical formulation lead to predictions.

The mathematical formulation often has inputs which are multivariate functions. A digital computer has only sets of numbers. Thus the computer has only partial information about the infinite-dimensional input.

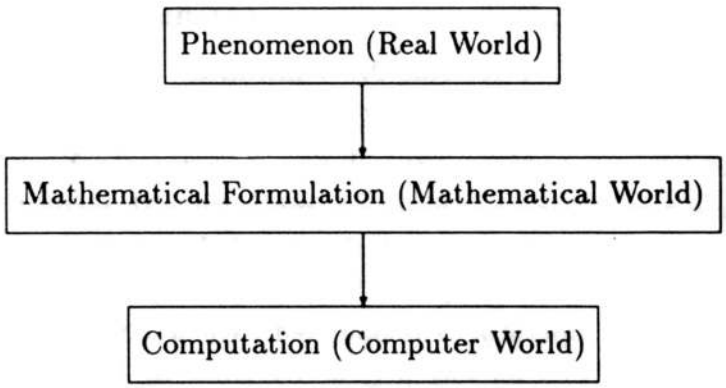

Figure 1. Three Worlds

Figure 2 schematizes the relations among the branches of computational complexity.

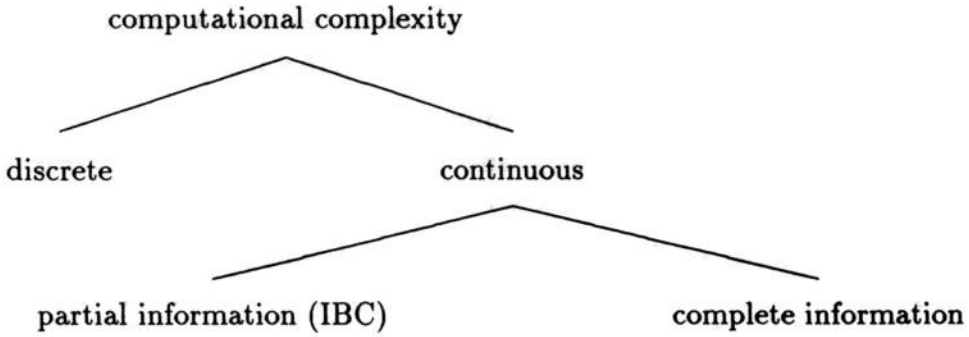

Figure 2. Structure of Computational Complexity

Figure 3 indicates the model of computation and information used by three branches of computational complexity.

	Discrete Combinatorial Complexity	Continuous Combinatorial Complexity	Information-based Complexity (IBC)
Model of Computation	Bit Model	Real Number Model	Real Number Model
Information	complete exact free	complete exact free	partial contaminated priced

Figure 3. Three Branches of Computational Complexity

Gaussian Mean Boundedness of Densely Defined Linear Operators

Nicholas N. Vakhania
Georgian Academy of Sciences, Tbilisi

It is known that a linear problem with solution operator $S : X \to Y$ in the probabilistic or average case setting has finite ε-complexity with respect to a probability measure μ iff $S \in L_2(X, \mu; Y)$ or, equivalently, iff $I \in L_2(Y, \mu \circ S^{-1}; Y)$ where I denotes the identity operator and $\mu \circ S^{-1}$ is the S-image of μ. If the measure μ is Gaussian and the linear operator S is bounded then $\mu \circ S^{-1}$ is also Gaussian and hence $I \in L_p(Y, \mu \circ S^{-1}; Y)$ for any $p \geq 0$. We show by two different approaches that this is the case also for linear unbounded densely defined Borel measurable S under the minimal natural condition $\mu(D(S)) = 1$ where $D(S)$ denotes the domain of S. Also we give the expression for the covariance operator of the transformed measure $\mu \circ S^{-1}$.

All Ill-Posed Problems are Solvable on the Average with Gaussian Measures

Arthur G. Werschulz
Fordham University and Columbia University, New York
(joint work with M. Kon and K. Ritter)

We wish to solve an ill-posed problem whose solution operator S is a measurable unbounded linear transformation. Let $\varepsilon > 0$. It is known that the ε-complexity of this problem is infinite in the worst case setting. Suppose we turn to an average case or probabilistic setting, the domain of S being equipped with a zero-mean Gaussian measure μ. It is known that the problem has finite ε-complexity iff $S \in L_2(\mu)$, and optimal information and algorithms are essentially the same as if S were bounded. We show that any such unbounded operator S belongs to $L_2(\mu)$. Hence, the ε-complexity of any such ill-posed problem is finite in the average case and probabilistic settings.

n-Widths of H^p-spaces in $L_q(-1,1)$

Klaus Wilderotter
Universität Bonn

The Kolmogorov n-width $d_n(H^p, L_q(-1,1))$ and the linear n-width $\delta_n(H^p, L_q(-1,1))$ of the Hardy space H^p in $L_q(-1,1)$ are asymptotically exact estimated. It is shown that for $1 \leq q < p \leq \infty$:

$$k_1\, n^{\frac{1}{2}(\frac{1}{q}-\frac{1}{p})} \exp(-\pi\sqrt{n/2\,(1/q-1/p)}) \leq d_n(H^p, L_q(-1,1))$$
$$\leq \delta_n(H^p, L_q(-1,1)) \leq k_2\, n^{\frac{1}{2q}} \exp(-\pi\sqrt{n/2\,(1/q-1/p)}).$$

This result is an decisive improvement of results of Burchard/Höllig: "n-Width and Entropy of H^p-classes in $L_q(-1,1)$", SIAM J. Math. Anal. **16**, 405–421, (1985), who proved the following weaker estimate for $1 \le q < p \le \infty$:

$$k_1\, n^{\alpha} \exp(-\pi\sqrt{n\,(1/q-1/p)}) \le d_n(H^p, L_q(-1,1))$$
$$\le \delta_n(H^p, L_q(-1,1)) \le k_2\, n^{\beta} \exp(-\pi/2\sqrt{n\,(1/q-1/p)}).$$

Average Case Complexity of Multivariate Problems

Henryk Woźniakowski
Columbia University, New York, and University of Warsaw

We study the average case complexity of approximating linear continuous operators defined on spaces of multivariate functions whose domain is $[0,1]^d$. The space is equipped with a Gaussian measure. We consider two classes of information: $\Lambda^{\rm all}$-all linear continuous functionals, $\Lambda^{\rm std}$-function evaluation. We say that the problem is tractable iff its ε-average case complexity is $O((1/\varepsilon)^p)$ with p independent of d. We show that tractability in $\Lambda^{\rm all}$ is equivalent to tractability in $\Lambda^{\rm std}$. Tractability in $\Lambda^{\rm all}$ is easy to check since we have only to check whether $\sum_{i=n+1}^{\infty} \lambda_i(d) = O((1/n)^{\alpha})$ for some positve α independent of d. Here $\lambda_i(d)$ are the ordered eigenvalues of the covariance operator of the Gaussian measure of solution elements. In this way we check positively the tractability of multivariate integration and approximation for Wiener sheet measures placed on mixed derivatives.

The use of relations between the average case on $C^{\vec{r}}([0,1]^d)$ and the worst case on $W^{\vec{r}+\vec{1}}([0,1]^d)$ enables to establish sharp complexity bounds for multivariate integration and approximation. Of particular importance are results from the worst case setting obtained by Temlyakov, and results from the average case setting on adaption obtained by Wasilkowski.

New Research Directions, Open Questions, Discussion

1. Large Linear Systems

Let $Ax = b$ denote a large linear system, where A is a sparse matrix. We want to compute x such that $\|Ax - b\| \leq \epsilon$ using only partial information about A.

(a) How many inner products are needed for certain classes of matrices for an ϵ-approximation?

(b) What is the complexity for certain classes of matrices if inner-product information is used?

Similar questions can be asked if we permit still more general information. Suppose we compute $L_i(A, b)$, where L_i denotes an arbitrary (adaptively chosen) linear functional on A and b.

(Traub)

2. Mean Value Arguments for Upper Bounds

For some problems the best upper bounds can be proved by abstract mean value arguments, i.e., the bounds are non-constructive. It is a challenge to prove constructive upper bounds in these cases. Several examples, mainly for multivariate integration, where given. Can computer experiments help to find optimal knots, at least for small n? Are Fibonacci-points optimal for $d = 2$?

(Niederreiter, Temlyakov, Woźniakowski)

3. Structural Complexity of Continuous Problems

In information-based complexity we often assume that simple arithmetical operations can be performed with unit cost. The assumption that also scalar multiplication and vector addition can be performed with unit cost may be problematic in some cases.

Using discretization one can change the model of computation such that all spaces are finite dimensional. Then one can give a new definition of combinatorial cost and obtains relations with other notions of complexity.

(Heinrich, Traub)

4. Exact Constants for Monte Carlo Methods

Using a new Minimax-Principle one can prove exact constants for Monte Carlo methods. Are similar results known for discrete problems? To which more general problems and algorithms (also non-continuous ones) can this Minimax-Principle be generalized?

(Mathé, Woźniakowski)

5. Monte Carlo Methods for Verification Problems
We know good Monte Carlo methods to test a matrix identity of the form $AB = C$. Study Monte Carlo methods for verification problems such as "Is A a singular matrix?"
(Karpinski)

6. Robustness of Zeros of Polynomials
Let P be a class of polynomials in two variables and let M be a subset of the complex plane. Find fast algorithms to test (studying only a few polynomials of P) whether all zeros of each $p \in P$ are elements of M. Several concrete examples for this general problem were given.
(Tempo)

7. Precomputing
We often assume that precomputation is allowed. Example: We prove that linear quadrature formulas $\sum_{i=1}^{n} c_i f(x_i)$ are optimal and simple in the sense that only $(n-1)$ additions and n multiplications are neccessary to apply such a formula. The (pre-) computation of the appropriate weigths c_i, however, possibly is very difficult. Sometimes it is interesting to analyze methods with the assumption that precomputation is not allowed. Several examples were given.
(Woźniakowski)

8. Which Measure?
What are appropriate measures for an average case analysis of numerical methods? Gaussian measures seem to be suitable in most cases. Sometimes it is reasonable to demand that the conditional probability measures should be nonvanishing.
(Kon and others)

9. For Which Problems Can Randomization Help?
We only have examples where randomization helps or does not help. There is no theory that characterizes for which problems randomization can help. What are the limits on how much randomization can help?
(Traub and others)

10. The Big F Problem
In many cases, the set F of problem elements is not known. Also the domain Ω and/or the probability measure μ might be unknown. Therefore one should study universal methods which are almost optimal under different assumptions. For which problems do such methods exist?
For many problems the definition of suitable stopping rules is difficult, for other problems one can find residual criteria.
(Traub and others)

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