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Computational Kinematics

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Report

of the Dagstuhl Seminar on

COMPUTATIONAL KINEMATICS

October 10th - October 14th, 1993

The aim of this seminar was to provide an account of the state of the art in *Computational Kinematics*. We understand here under this term that branch of kinematics research involving intensive computations not only of the numerical type, but also of a symbolic nature.

Research in kinematics over the last decade has been intensive in the computational aspects of kinematics problems. In fact, this work has been prompted by the need to answer fundamental questions such as the number of solutions, whether real or complex, that a given problem can admit. Problems of this kind occur frequently in the analysis and synthesis of kinematic chains, when finite displacements are considered. The associated models, that are derived from kinematic relations known as *closure equations*, lead to systems of nonlinear *algebraic* equations in the variables or parameters sought. What we mean by algebraic equations here is equations whereby the unknowns are numbers, as opposed to differential equations, where the unknowns are functions. The algebraic equations at hand can take on the form of multivariate polynomials or may involve trigonometric functions of unknown angles.

Because of the nonlinear nature of the underlying kinematic models, purely numerical methods turn out to be too restrictive, for they involve iterative procedures whose convergence cannot, in general, be guaranteed. Additionally, when these methods converge, they do so to only isolated solutions, and the question as to the number of solutions to expect still remains. These drawbacks have been overcome with the development of continuation techniques that are meant to produce all solutions to a given problem. While continuation technqiues have provided solutions to a number of problems, they are still subjected to the uncertainties of iterative techniques. Hence, alternative approaches have been sought, that rely on modern software and hardware for symbolic computations. Commercial software of this kind is now very reliable and widespread; it has naturally found its way into kinematics research. In fact, current research in kinematics involves symbolic manipulations that were impossible to even imagine as recently as fifteen years ago, when the first symbolic manipulation packages started coming out of the computer science laboratories. Among other techniques, resultant methods based on dyalitic elimination were discussed critically at the seminar, while Gröbner bases were proposed as a powerful elimination technique .

The seminar covered trends and progress attained in Computational Kinematics in a broad class of problems. The workshop topics were divided into seven parts, namely, *i*) kinematics algorithms, whereby general kinematics problems are discussed in light of their solution algorithms; *ii*) redundant manipulators, which is self-descriptive; *iii*) kinematic and dynamic control, in which the link between kinematics and the disciplines of dynamics and control is established;

iv) parallel manipulators, self-descirptive as well; *v*) motion planning, touching upon computational geometry; *vi*) kinematics of mechanisms, in which the main issue is the presence of closed kinematic chains; and *vii*) a special section of miscellaneous contributions that were scheduled after the original program was mailed to the participants.

All contributions to the seminar, except those of the special section were published under the title "Computational Kinematics" in volume 28 of the series "Solid Mechanics and its applications" by Kluwer Academic Publishers, Dordrecht.

IBFI Schloß Dagstuhl is herewith given due acknowledgement for its financial and logistical support and encouragement. This support made it possible to bring together specialists of various disciplines working in the area. Among the participants, who met for one week at Schloß Dagstuhl, we count engineers, computer scientists and biomechanicists, all of whom share a common interest, namely, Computational Kinematics. Prof. Dr. Reinhard Wilhelm, Scientific Director of IBFI, and his staff are especially acknowledged for their support and their outstanding hospitality that contributed significantly to the success of the workshop.

Jorge Angeles, Günter Hommel and Peter Kovács

Kinematics Algorithms

COMPUTATIONS IN KINEMATICS

B. Roth

Several methods to solve sets of nonlinear equations are discussed. Then a modification of dialytic elimination is described, and two methods for obtaining new linearly independent equations are presented. Finally, one of the methods is applied to three quadratics and is shown to yield all of the solutions, without any extraneous roots.

REDUCING THE INVERSE KINEMATICS OF MANIPULATORS TO THE SOLUTION OF A GENERALIZED EIGENPROBLEM

M. Ghazvini

An eigenproblem-based method is presented which reduces the full determination of the inverse kinematics of manipulators with revolute and prismatic joints, and of related single-loop mechanisms, to the solution of a generalized eigenproblem. It is shown how to obtain directly an eigenproblem of minimal dimension, i.e. no extraneous eigenvalues appear at any step of the method. Therefore, no extra work is neccessary to eliminate extraneous eigenvalues (roots). Such, applying the here presented method to the general 6R-manipulator about 40% less computations have to be performed. The eigenproblem-based methods are numerically more stable and accurate than polynomial-based methods, because eigensystems can be computed without the previous, numerically ill-conditioned determination of the characteristic polynomial. Furthermore, a compact, well structured and efficient formulation of the basic equations is shown. Such, the basic equations are determined directly without further expensive symbolic simplifications. Numerical results are reported at the end.

ON THE TANGENT-HALF-ANGLE SUBSTITUTION

P. Kovács and G. Hommel

The tangent-half-angle substitution is commonly used to convert goniometric equations in the sine and cosine of a certain variable θ into polynomial equations in a new variable $x = \tan(\theta/2)$. This facilitates the solution of goniometric equations and systems of equations. Elementary problems concerning the special case $\tan(\theta/2)$ and the introduction of trivial extraneous roots into systems of equations are well known and can be handled. The article shows that nontrivial extraneous roots may be generated when a tangent-half-angle substitution for some variable is used to derive a characteristic equation for another variable from a goniometric system of equations. An effective method is presented to decide before the symbolic solution of a system whether a tangent-half-angle substitution produces such extraneous roots. The results can also be used to detect relevant simple subclasses of manipulators in a given superclass when no general symbolic solution for the superclass is explicitly known.

As an application, it is proven that the Raghavan-Roth algorithm for the symbolic solution of the inverse kinematics problem never generates these nontrivial extraneous roots.

RESULTANT METHODS FOR THE INVERSE KINEMATICS PROBLEM

J. Weiss

We closely reconsider the mathematical tools needed for the solution of the inverse kinematics problem for 6R series manipulators by resultant methods. We discuss the reduction of the original problem, the homogenization of the reduced equations, and several approaches for the application of resultant methods.

Redundant Manipulators

REDUNDANCY RESOLUTION FOR AN EIGHT-AXIS MANIPULATOR

H. Heiß

A method allowing closed form solutions of the joint variables (inverse kinematic problem) has been developed for a man-like kinematic structure. Moreover, the procedure takes into account the available joint range, thus opening up new ways towards intelligent optimization strategies.

A MIXED NUMERIC AND SYMBOLIC APPROACH TO REDUNDANT MANIPULATORS

M. Kauschke

This paper presents a method for generating locally optimized link variable solution for redundant serial link robots. It combines closed form solutions for the inverse kinematic problem of nonredundant subchains and a numeric approach solving the local optimizing problem. The result is an efficient method for low degree of redundancy and complex optimizing tasks.

COMPUTATIONAL CONSIDERATIONS ON KINEMATICS INVERSION OF MULTI-LINK REDUNDANT MANIPULATORS

J. Lenarčič

In this work, the pseudoinverse-based methods are evaluated for the kinematics inversion of hyper-redundant multi-link robot manipulators. The validity of these methods is questioned from the viewpoint of the computational efficiency specified in terms of arithmetic operations per iteration step. It is shown that less computation time is needed by steepest descent methods especially when the initial estimation is far from the solution.

ON FINDING THE SET OF INVERSE KINEMATIC SOLUTIONS FOR REDUNDANT MANIPULATORS

E. Celaya and C. Torras

Given a serial manipulator with n d.o.f. and redundancy ρ, the set of solution vectors $q = q_1..q_n$ to the inverse kinematic equation $q = f^{-1}(x)$ for a regular end-effector pose x is a set S of dimension ρ, and can be interpreted as the configuration space of an associated kinematic loop with mobility $M = \rho$. A recursive way to describe this set S is introduced, which is based on the exact computation of the sets S_i of values allowed for each joint variable q_i in the loop, corresponding to the projections of the solution set S onto the axes.

The description of the solution set S at the highest level provides the minimum box (or a number of separate boxes) with edges parallel to the axes containing S. Recursively more accurate descriptions of S may be obtained by fixing one variable q_i to any value within its allowed set S_i and computing the sets of allowed values for the remaining variables in a new loop. Each step in this process can be visualized as 'slicing' the set by a hyperplane of constant q_i and finding the minimum box containing the solution restricted to this hyperplane.

This description permits including in a straightforward way the physical limitations that usually exist in actual joints, and is amenable to the solution of multiloop problems that arise from parallel manipulators. Explicit expressions for the allowed sets of values for variables in all planar and spherical loops are provided.

Some considerations based on the form that such sets can take permit the establishment of criteria to assess topological properties of the solution set as a whole, such as the number of self-motion manifolds and the existence or not of cranks (fully rotatable joint variables).

THE SELF-MOTION MANIFOLDS OF THE N-BAR MECHANISM

F. Thomas

This paper investigates the global sets of solutions for single loop inverse kinematic problems containing only independent rotational and translational degrees of freedom. Since any kinematic loop can be modeled as a loop equation derived from the n-bar mechanism, we concentrate on the study of this particular mechanism.

The set of solutions for the redundant orthogonal spherical mechanism can be stratified leading to several manifolds (called self-motions manifolds). It is shown that the solution to the translational degrees of freedom of the n-bar mechanism is provided by the tangent bundle of these manifolds. Then, we conclude that the information required for the analysis of any single loop spatial mechanism is essentially stored in the self-motion manifolds of the orthogonal spherical mechanism. A simple set of local charts that provide an atlas for these manifolds is also discussed. Finally, two examples that illustrate the main ideas are presented.

Kinematic and Dynamic Control

NONLINEAR CONTROL OF CONSTRAINED REDUNDANT MANIPULATORS

C. Woernle

The dynamic hybrid position and contact force control of open-chain manipulators with redundant degrees of freedom by means of exact input-output linearization is investigated. The dynamics of the manipulator is transferred into a decoupled linear dynamics in the subspace of the control variables and a still nonlinear dynamics in the subspace of the redundant degrees of freedom. By projecting artificial potential and damping forces into the subspace of the redundant degrees of freedom, it is possible e.g. to realize repeatable motions in the joint space for cyclic trajectories or to avoid collisions with obstacles in the workspace.

ANALYSIS OF MECHANISMS BY THE DUAL INERTIA OPERATOR

M. Shoham and V. Brodsky

The application of dual numbers to kinematics is based on the principle of transference that extends vector algebra to dual vector (motor) algebra. No such direct extension exists however, for dynamics. Inertia binor is used to obtain the dual momentum, from which the dual equations of motion are derived. This derivation raises the dual dynamic equations to six dimensions, and in fact, it does not act on the dual vector as a whole, but rather on its real and dual parts as two distinct real vectors.

In this investigation, the dual inertia operator is introduced. This gives the mass a dual property which has the inverse sense of Clifford's dual unit, namely, it reduces a motor to a rotor proportional to the vector part of the former, allowing direct relation of dual force to dual acceleration. As a result, the same equation of momentum which holds for linear motion, holds also for angular motion if dual force, dual acceleration, and dual inertia, replace their real counterparts.

This approach was implemented in a symbolic computer program. By adding dual number algebra, the program is able to handle dual quantities. Furthermore, applying the dual inertia rules, the dual equations of motion are obtained by replacing real with dual quantities as it is illustrated in the example of a three-degrees-of-freedom robot.

Parallel Manipulators

DIRECT KINEMATICS IN ANALYTICAL FORM OF A GENERAL GEOMETRY 5-4 FULLY-PARALLEL MANIPULATOR

C. Innocenti and V. Parenti-Castelli

This paper presents the analytical form solution of the direct position analysis for a fully-parallel manipulator that features a base and a platform connected by six adjustable-length legs whose extremities meet the base and the platform respectively at four and five points. When the leg lengths are given, the manipulator becomes a statically determined structure that can be assembled in different configurations. The direct position analysis aims at solving all possible configurations. In the paper, the analysis is first reduced to the solution of a three non-linear equation

system in three unknowns, then two unwanted unknowns are eliminated thus obtaining a final 24th order polynomial equation in only one unknown. The twenty-four roots of the equation provide as many configurations of the 5-4 structure in the complex field. Numerical examples support the new theoretical findings.

THE KINEMATICS OF 3-DOF PLANAR AND SPHERICAL DOUBLE-TRIANGULAR PARALLEL MANIPULATORS

H. R. Mohammadi Daniali, P. J. Zsombor-Murray and J. Angeles

Two double-triangular mechanisms are introduced here. These are planar and spherical three-degree-of-freedom mechanisms that consist of two triangles moving with respect to each other. Moreover, each side of the moving triangle intersects one corresponding side of the fixed one at a given point defined over this side. The direct kinematic analysis of the mechanisms leads to a quadratic equation for the planar and a polynomial of 16th degree for the spherical mechanism. Numerical examples are included that admit two real solutions for the former and four real solutions for the latter, among which only two positive values are acceptable. All solutions, both real and complex, are listed.

THE SEMIGRAPHICAL SOLUTION OF THE DIRECT KINEMATICS OF GENERAL PLATFORM-TYPE PARALLEL MANIPULATORS

K. E. Zanganeh and J. Angeles

A semigraphical method is presented for computing all real direct kinematic solutions of platform-type parallel manipulators with general geometries. The direct kinematic problem is reduced to basically two bivariate equations in the sines and cosines of two unknown angles. One equation is derived by solving an overdetermined system of equations that can be perturbed by different multiples of the least-square error involved in the solutions. Upon perturbing this equation by two different multiples, two distinct equations are obtained. The first bivariate equation and each of these two equations define three contours in the plane of the two angles involved, the intersections of these contours providing all real solutions. The method is used to find all real direct kinematic solutions of a general parallel manipulator of the platform type.

On the Representation of Rigid-Body Motions and its Application to Generalized Platform Manipulators

D. Lazard

Different ways for representing rigid-body motions (direct isometries) in a computer are presented. It appears that the choice between them may have a dramatic effect on the difficulty of a computation or of a proof.

As an application, a computational proof is given of the fact that the direct kinematic problem for a generalized Stewart platform has at most 40 complex solutions.

Algebraic-Geometry Tools for the Study of Kinematics of Parallel Manipulators

J.-P. Merlet

Manipulation of algebraic equations arise frequently in kinematic problems. But in many of these problems it is not necessary to solve the algebraic equations to establish interesting results as sometimes only the number of real solutions is important. Fortunately many theorems in algebraic geometry, some of them being not well known, may give some insight on this point. We present some of these theorems and show how they can be applied to demonstrate interesting results in the field of kinematic problems for parallel manipulators.

Motion Planning

Singularity Control for Simple Manipulators using "Path Energy"

J. Lloyd

A new method is presented for controlling the trajectories of straight-line Cartesian paths near the kinematic singularities of simple manipulators. Conventional approaches to this problem, which typically employ a pseudo-inverse of the manipulator Jacobian, result in path deviations and have difficulty controlling joint accelerations. The more global approach described here

uses a "potential function" in the region of the singularity to reduce the path velocity in such a way that both the joint velocities and accelerations remain bounded without incurring any deviation from the desired path. For cases such as the elbow or shoulder singularities of the PUMA manipulator, the results are very good and the necessary computations are simple enough to be done on-line.

AN INVESTIGATION OF PATH TRACKING SINGULARITIES FOR PLANAR 2R MANIPULATORS

J. Kieffer and B. O'Loghlin

The planar 2R manipulator is used as a vehicle for investigating the problem of tracking end-effector paths that force the manipulator into a singular configuration. Results show that isolated points, turning points, nodes, cusps, hypernodes, and hypercusps can arise in the locus of inverse kinematic solutions depending the end-point path's degree of contact with the workspace boundary. Methods for determining smooth local representations of each type of path-tracking singularity are developed based on low-order analysis. These representations provide complete low-order information on all families of trajectories that track the path at the singularity.

ROBOT MOTIONS WITH TRAJECTORY INTERPOLATION AND OVERCORRECTION

H. Heiß

This paper makes proposals for trajectory interpolation in joint space and in cartesian space regarding as well orientation as position and enlarging the application bandwidth; furthermore it shows new methods for connecting path segments in a smooth manner, which take into account not only velocity but also acceleration values and improve the facility of robot programming by user-defined starting and end points of the smoothing interval.

COMPUTATIONAL GEOMETRY AND MOTION APPROXIMATION

Q. J. Ge and B. Ravani

This paper develops a geometric construction algorithm for desiginig a second order geometrically (G^2) continuous motion. It combines results in kinematics with the notion of geometric continuity from the field of Computer Aided Geometric Design and develops geometric conditions for piecing two motion segments smoothly. A complete algorithm is presented for con-

structing a G^2 continuous piecewise Bezier type motion. The results have applications in mechanical systems animation, computer vision, robot trajectory planning and key framing in computer graphics.

Kinematics of Mechanisms

FORWARD KINEMATICS OF A 3-DOF VARIABLE-GEOMETRY-TRUSS MANIPULATOR

R. Hertz

The forward kinematics of an octahedral type variable-geometry-truss manipulator is presented. The manipulator is comprised of two stacked octahedral trusses. The intersection of the octahedra consist of 3 linear actuators, which are used to control the position of the moving plane of the manipulator relative to the base, giving the mechanism a 3-DOF capability. A kinematical model of the manipulator is presented which includes important features of non-equal octahedral geometry and inter-hinge displacements. Vectorial equations are formulated to reduce the problem into two systems of nonlinear equations, each of which has three unknowns. Each system of equations is shown to have 8 reflected solution pairs through the formulation of polynomials in one variable, giving the manipulator a maximum of 256 unique configurations for one set of actuator lengths. Numerical examples confirm the validity of the results.

ANALYTICAL DETERMINATION OF THE INTERSECTIONS OF TWO COUPLER-POINT CURVES GENERATED BY TWO FOUR-BAR LINKAGES

C. Innocenti

The paper presents the analytical determination of the intersections of two coupler-point curves generated by two distinct four-bar linkages. After devising a suitable set of three compatibility equations, and performing algebraic elimination, a final polynomial equation of eighteenth order with only one unknown is obtained whose roots represent the sought-for intersections. As a result, two coupler-point curves cross each other, in the complex field, at eighteen points. The contribution is aimed at further delineating the analytical properties of four-bar linkage coupler-point curves. A case study is finally reported.

On Closed Form Solutions of Multiple-Loop Mechanisms

A. Kecskeméthy

Discussed in this paper is a method for the automatic generation of symbolic equations for multiple-loop mechanisms whose kinematics can be solved in closed form. The method is based on geometric and topological properties of the system which are invariant to coordinate transformations. It focuses on the treatment of the individual multibody loops as transmission elements which encompass the solution of the local nonlinear constraint equations and which can be assembled by linear equations to yield general mechanisms. The global processing is obtained as a combination of algorithms for generating the local kinematics of the individual loops, detecting a suitable set of independent loops, and finding an optimal "solution flow" in the resulting kinematical block-diagram which represents the order in which the equations are to be solved. An example processed by the current implementation of the method with the symbolic-computation language Mathematica illustrates the basic ideas and the scope of the approach.

A Modular Method for Computational Kinematics

P. Fanghella and C. Galletti

A modular method for symbolic kinematic modelling of multiloop mechanisms is outlined. For a given mechanism, the method identifies automatically a list of modules for which closure equations can be generated and solved hierarchically. Closed form solutions can be obtained in many cases of practical interest.

Synthesis for Rigid Body Guidance Using Polynomials

A. A. Rojas Salgado, T. Navarro and J. Isidro

A method for the exact synthesis for rigid-body guiding mechanisms through polynomials with dimensional constraints is presented. This method produces a broad solution spectrum that can be visually examined. The designer can choose a defect-free mechanism within a restricted area. The search along perpendicular axes by means of polynomials yields thorough solutions. An example is presented for illustration purposes.

DESIGNING MECHANISMS FOR WORKSPACE FIT

F. Park, A. P. Murray and J. M. McCarthy

In this paper we examine the problem of designing a mechanism such that its tool frame comes closest to reaching a set of desired goal frames. We regard SE(3), the Euclidean group of rigid-body motions, as a Lie group, and demonstrate that a left-invariant Riemannian metric on SE(3) provides not only a solution that is left-invariant with respect to choice of inertial frame, but also a natural means of regarding SE(3) as a metric space. To illustrate our methodology this metric is applied to the design and positioning of certain planar and spherical mechanisms.

Special Session

ON SOME PROBLEMS CONCERNING EQUIDISTANCE FROM LINES

M. L. Husty, H. Sachs and P. J. Zsombor-Murray

Two equidistance problems are investigated. At first the line-line equidistance, where a line, equidistant to two given lines g_1, g_2 has the same distance and the same angle to both lines. Given a third line g_3 then it has been shown [1] that there exists one and only one line equidistant to all three given lines. If g_3 moves with an one parameter motion, then the equidistant line describes a ruled surface. It is shown that if the motion is a rotation of g_3 around g_1, then the equidistant surface is a ruled surface of order six. A fourth line g_4 rotating around g_2 describes another ruled surface of order six. The main theorem shows that if g_1, g_2, g_3 and g_4 form the revolute axes of a Bennett mechanism, then the two equidistance surfaces coincide.

In the special case where all angles and all distances of the Bennett mechanism are equal, the two equidistance surfaces are coincident hyperbolic paraboloids.

In the second part, point-line equidistance is considered. Recalling the result that all points equidistant to two lines are on an orthogonal hyperbolic paraboloid, it is shown that for four given lines eight equidistant points exist in general. These eight points can be found by intersecting three equidistance paraboloids. Then the eight solution points are considered as centeres of spheres which touch all four given lines. An example is given where all eight spheres are real.

[1] BOTTEMA,O.-ROTH,B.: Theoretical Kinematics, North Holland Publishing Company, Amsterdam - New York - Oxford, 1979.

SINGULAR CONFIGURATIONS AND CLOSED KINEMATIC CHAINS

A.Karger

In this contribution we shall describe the connection between singular configurations of robot manipulators and movability of closed kinematic chains. A justification for this question lies in the fact that closed kinematic chains can have freedom of motion only if its axes are in a singular configuration. This condition is only necessary, but we can show that under special circumstances it is also sufficient. We consider robot manipulators with rotational links only. At first we show that a robot-manipulator with all configurations singular can be closed to a kinematic chain with a freedom of motion at each of its configurations and such robot-manipulators are explicitly given. Apart from those robot-manipulators we have the following statements for 4 and 5 parametric robot-manipulators: If a 4-parametric robot-manipulator has infinitely many singular configurations, then it can be closed to a chain with one degree of freedom and it is the Benett's mechanism. If a 5-parametric robot-manipulator has a 2-dimensional singular set, then it leads to Goldberg linkages similarly as in the previous case.

AUTOMATIC SYMBOLIC MODELLING OF ROBOT KINEMATICS

W. Khalil

This paper presents the automatic symbolic modelling of robots through the software package SYMORO+ developed by the robotics team of L.A.N. This package permits to generate the direct and inverse geometric and kinematic models, the direct and inverse dynamic model, and the inertial identification model. The package runs on Sun stations, the modelling programs are written in MATHEMATICA language, while the interface program and an optimizer program are written in C language.

The package can handle open loop robots, tree structure robots and closed loop robots, the description of the robots is carried out using Khalil and Kleinfinger notations which has been presented at the IEEE Robotics and Automation conference 1986.

In the inverse geometric modelling the methods of Paul, Pieper and the general method of Raghavan and Roth have been adapted for this notation. In the dynamic models the customized calculation, the use of the base inertial parameters, and the use of the program "Optimizer" lead to get models with reduced number of operations

APPLICATION OF THE GROUP AND GEOMETRICAL PROPERTIES OF E(3) TO THE PROBLEMS OF RIGID-BODY SYSTEMS

C. Mladenova

This paper treats the kinematical problems in describing the Euclidean motions of rigid-body systems. In the analytic representation, the rotation is expressed by defining its action on a vector and the Lie group SO(3) is parametrized by vector-parameters making also a Lie group with a clear geometrical sense and a simple composition law. Because of the fact that every Euclidean motion may be presented as a screw motion, the paper proposes a useful interplay of screw geometry, dual algebra and vector and matrix transformations. The special structure of a manipulator as a series of coupled bodies allows the specialization of the general line coordinate transformation matrix in the form called dual orthogonal matrix and defining of the notion dual vector-parameter. The geometrical and kinematical models of a manipulator are expressed in a closed form using dual orthogonal matrices and dual vector-parameters. The angular velocity of the end-effector takes a particularly simple form leading to a simple geometric interpretation of the Jacobian matrix.

It is worth mentioning here, that this paper is a continuation of previous investigations of the author, where the kinematics, dynamics and control of robot manipulators are considered in vector-parameter configurational space. Because of the fact that only one of the components of the vector-parameter is informative, the dimension of the system is not increased and the number of the elementary computations in the direct kinematic problem, forming of the Jacobian matrix, dynamic modeling and full simulation of the motion is reduced with about 40% in comparison with the methods used till now. The kinematic and dynamic differential equations are pure algebraic equations over a Lie group.

PRACTICAL CONSIDERATIONS FOR THE DESIGN OF PARALLEL ROBOTS

F. Pierrot

The lecture addresses problems encountered when designing and building parallel robots. The presentation is essentially based on the collaboration between two Academic Research Groups (namely, LIRMM, in Montpellier, France, and Tohoku University, Sendai, Japan) and a Robots Manufacturer (namely, Toyoda Machine Works, ltd., Kariya, Japan). The answer to the basic question "Why designing parallel robots" came from anlysis of serial robots major drawbacks: bad stiffness, poor accuracy, limited velocity and acceleration ...

We then decided to create a 6 dof high-speed parallel robot to tackle with high-speed complex tasks, such as fast insertion tasks: the HEXA architecture was born. The first step of the design was to solve both inverse and direct kinematics. Due to the parallel arrangement, it was obvious to derive the inverse kinematics in analytical form, and impossible to derive the direct kinematics in the same way: we used numerical iterative methods. With these two series of equations, we were able to do an kinematic optimization. We sized the robot (in term of link lenghts) in order to obtain the best solution to the following problem: avoiding the so-called "overmobility position" (i.e.: the position where the robot stiffness becomes zero!). However, we did this optimization under some very important practical constraints due to various technological choices (actuators, passive joints, mechanical transmission system ...).

Finally, we obtained a new structure that is able to produce 6 m/s velocity, 22 g acceleration and is able to insert a peg in a hole in less than 0.1 second with a clearance smaller than 10 microns. However, one problem is still open: what can a parallel robot dedicated CAD package be?

Topics of the Open-Problem Session

Calibration of parallel manipulators (J-P. Merlet)

Let us consider two rigid bodies connected by 6 extensible links, which have an extremity on each of the bodies. Suppose you can measure the position / orientation of one of the bodies with respect to a frame attached to the other body and the corresponding lengths of the links. The problem of calibration is to use these measurements to determine the coordinates of the attachment points of the links to the body. It seems that for a given set of measurements the solution set is not unique. Devise a method to find the unique solution.

Near-simple kinematics (G. Hommel)

When a given manipulator type with simple kinematical structure is manufactured industrially, the dimensions of each produced "real" manipulator usually differ from those of the desired "ideal" one by small parameter variations. Such manipulator geometries are called "near-simple". The actual parameters of each of the real manipulators can be identified exactly by todays calibration techniques. The effort required for an "ordinary" symbolic solution of the inverse kinematics of near-simple manipulators comes close to solving the kinematics of general manipulators with arbitrary dimensions since none of the link parameters (Denavit-Hartenberg parameters) can be assumed to be trivial. On the other hand it is obvious that solutions deviate only marginally from the solutions of the ideal robot since tolerances can be assumed to be small.

General manipulator geometries can be solved symbolically with the well known Raghavan-Roth-Algorithm or its fastest available acceleration at the moment by M Ghazvini (see this report). Numerical methods can also be used to solve the problem (see: Hayati, S.A., Roston, G.P. *Inverse Kinematic Solution for Near-Simple Robots and its Application to Robot-Calibration*. In: Recent Trends in Robotics: Modelling, Control and Education (Eds.: Jamshidi, Luh, Sahinpoor), Elsevier Science Publishing Co., 1986). However, all these techniques may still be too slow for todays robot control units which are usually equipped with relatively simple hardware. The question is whether new theoretical methods can be found to solve the inverse kinematics of near-simple manipulators with less effort. Any such method should exploit the knowledge of the symbolic inverse kinematic solution of the simple, ideal manipulator.

B-spline motion interpolants (B. Ravani)

The extension of motion approximation and interpolation to include B-spline motions has to be investigated.

"Semi-simple" solutions and mobility analysis for multiple-loop mechanisms (A. Kecskeméthy)

While there exist many methods for the symbolical resolution of the inverse kinematics of a single loop, the multiple-loop, spatial case has not been treated as thoroughfully, and also the very complex open question of the mobility of non-trivial mechanisms, as the ‚paradoxical' cases described in Jorge Angeles's book "Rational Kinematics", has not been pursued very far. The proposition here is to investigate these cases from a topological and geometrical viewpoint, and to derive methods both for finding closed-form solutions, and to detect over-constrained, but movable substructures in the general case. The results may be quite rewarding, because the multiple-loop case is very common in modern mechatronic systems, and thus the same efficient and well-understood methods will be needed soon for multiple-loop mechanisms as in the decades before for the single-loop, i.e., the "robotics" case.

Is the direct kinematics problem solved? (C. Innocenti)

Find a polynomial solution to the direct kinematics of the general-geometry 6-6 fully parallel manipulator. Such a solution would represent an alternative, more efficient approach to the determination of all closure configurations of the 6-6 manipulator, now accessible only via continuation methods. From an operative standpoint, it would suffice to numerically evaluate, with arbitrarily high accuracy, all coefficients of the involved 40-th order algebraic equation: determination of all assembly configurations will directly ensue.

Degree of kinematic redundancy with and without joint limits (J. Lenarčič)

In different regions of the workspace a non-redundant mechanism can achieve different numbers of solutions of the inverse kinematics problem, i.e. different number of configurations in the same Cartesian point of the end-effector. This depends on the kinematic structure of the mechanism, link lenghts, twist angles, and joint limits.

The objective would be to expand the regions in which the mechanism can achieve the maximum number of solutions since this gives more flexibility in solving a given task. The problem is to efficiently calculate the regions of the workspace with different numbers of solutions, as well as to optimise the mechanism in order to maximise the mentioned flexibility.

A more complex related problem is to develop an approach to treat the redundant mechanisms in the same way. This would serve not only to optimise a given mechanism but also to distinguish between different redundant mechanisms and would enable to "quantify" the redundancy. The problem can be considered as a generalisation of the problem of calculating the ranges of the mechanism's self-motion.

Complete enumeration of all manipulators with a quadratic inverse kinematic solution (P. Kovács)

The most relevant manipulators with respect to industrial applications are those with a "quadratic symbolic solution", meaning that all joint configurations for any given effector pose can be found by solving at most quadratic equations. Large manipulator classes with quadratic solutions are known. It would obviously be very interesting to determine the *complete* class of these manipulators. It is questionable if existing theoretical methods are sufficient to solve this problem. It may be possible to identify additional "quadratic classes" with limited effort. However, the more difficult task probably consists in *proving* the *completeness* of the obtained family of quadratic manipulators.

Dagstuhl-Seminar 9341

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