Fourth Dagstuhl Workshop on Geometric Modelling

16.05 - 21.05.1999

organized by

Hanspeter Bieri, Universität Bern, Switzerland
Guido Brunnet, Universität Kaiserslautern, Germany
Gerald Farin, Arizona State University, Tempe USA

Geometric Modeling is that field within Computer Science which is concerned with the efficient processing of geometric information on a computer. It combines techniques from Computer Science, Mathematics and Engineering and attracts researchers with different scientific backround. Application areas of Geometric Modeling include CAD/CAM, Computer Graphics, Medical Imaging and Scientific Visualization.
The fourth Dagstuhl seminar on Geometric Modeling was attended by 56 participants from 15 different countries. Among these the leading experts of the field were present as well as a several young researchers who received financial support from the European TMR-programm. For the second time during a Dagstuhl seminar the John A. Gregory Memorial Award was awarded to the pioneers of the field of Geometric Modeling.
The talks presented at the conference concerned the problem areas of

- Reverse Engineering
- Geometric Models for scientific visualization and simulation
- Multiresolution methods for complex geometric objects
- Variational Design
- Geometric data structures
- Error analysis of geometric operations.

The following collection of abstracts gives an impression of the wide range of topics that were dicussed.

Contents

1 The Extreme Vertices Model (EVM) for Orthogonal Polyhedra 4

2 Muti-resolution Reconstruction of Smooth Fat Surfaces 4

3 Mesh based anisotropic energies for free form surfaces 5

4 Nef Polyhedra and Selective Geometric Complexes 5

5 Multidimensional Geometric Modelling and Variational Design 6

6 A Declarative Modelling System 6

7 Direct Segmentation of Unorganized Point Sets Based on Nearest Neighbor Criteria 7

8 Light-Field Modelling 7

9 Rational Bisectors of Rational Varieties in R^3 8

10 A Permanence Principle for Shape Control 8

11 Wavelets on Triangulations 9

12 Blossoming, the Cancellation Axiom, and Divided Difference 9

13 Applications of Contouring 10

14 A quintic G^1 triangular interpolating spline 10

15 Integral spline surfaces with rational offsets 11

16 C-continous Shape-Preserving Interpolants of Variable Degree 11

17 Topological Techniques in Intersecting Surfaces of Special Types 11

18 Voronoi Diagrams and Strategies for Autonomous Robots 12

19 Multiresolution Modeling on Meshes 13

20 Triangulating 3D Surfaces from Unorganized Points 13

21 Simulated Annealing and the Construction of Linear Spline Approximations to Scattered Data 14

22 Theory and algorithms for non-uniform spline wavelets 15

23 Cylindrical Surface Pasting **15**

24 Low-Discrepancy Sequences for Computing Volume Properties **16**

25 An affine invariant norm based upon the Loewner ellipsoid **16**

26 Implicit Surface Approximation Using Adaptive Spatial Meshes **17**

27 Discrete Algorithms for Qudrilateral and Hexahedral Mesh Generation **18**

28 Exploiting Wavelet Coefficients for Modifying Functions **18**

29 Control net approximation to splines **19**

30 Error Propagation in Geometric Constructions **20**

31 A Shape Modelling API for the STEP Standard **20**

32 Convexity Preserving Interpolation **21**

33 Best Bounds on the Approximation of Splines by their control net **22**

34 Recent Advances in Macro-Element Methods for Fitting Scattered Data **22**

35 Implicit Surfaces Revisited - the I-patches **23**

36 Subdivision Schemes for Flow **24**

37 Shape Parametrisation for Biomedical Applications **24**

38 Analytical Concepts and Tools for Computing Geodesic Voronoi Diagrams and Geodesic Medial Axes on Free Form Surfaces **25**

39 Applications of Perturbation Theory to B-Spline Interpolation and Approximation **26**

1 The Extreme Vertices Model (EVM) for Orthogonal Polyhedra

Dolors Ayala and A. Aguilera, Universitat Politecnica de Catalunya

This work deals with the specific type of Orthogonal Polyhedra. First, the Extreme Vertices Model (EVM) for this kind of objects is presented. This model turns out to be very concise but is a complete solid model. The formal properties of this model are also discussed. Then an algorithm to convert from this model into a hierarchical Boundary Representation model is described. Although the completeness of the model is proved formally, this algorithm shows how all the information that is not explicitly stored in the model comes out. Finally, the Alternating Sum of Volumes decomposition is applied to this particular type of polyhedra by taking profit of the simplicity of the presented model.

2 Muti-resolution Reconstruction of Smooth Fat Surfaces

Chandrajit L. Bajaj and Guoliang Xu, University of Texas at Austin

Several naturally occurring as well as manufactured objects have shell like structures, that is the boundary consists of surfaces with thickness. In an earlier paper, we have provided a reconstruction algorithm for such shell structures using smooth fat surfaces within three-sided prisms. In this paper, we extend the approach to a scaffolding consisting of three and four-sided prisms. Withing each prism the re-constructed function is converted to a rational Bezier representation. In addition to the adaptive feature of our earlier scheme, the new scheme has the following extensions:

(a) the size of individual fat patches are much bigger;

(b) the re-constructed fat function has a multi-resolution representation;

(c) the error of approximation is ϵ bounded for any given ϵ;

(d) fairing techniques are combined to obtain nice shaped fat surfaces.

3 Mesh based anisotropic energies for free form surfaces

M. Bercovier, T. Matskewich, O. Volpin, Hebrew University of Jerusalem

Mesh based anisotropic energies for free form surfaces and mesh based isotropic formulation (Greiner) are compared .Both take into account the mesh geometry (first order scaling). Important differences: we introduce a plate type energy and separate the in plane vector field and the normal one. Hence the Bezier patches and the correponding energy functionnal are defined relatively to the plane of the underlying quadrilateral:we have a decomposition in normal and in plane displacements , hence we follow the geometry Resulting behaviour : Greiner's approach will lead for first order energies, minimal surface like shapes, ours will follow the underlying mesh. Behavior near singular situations will be different . This is illustrted in the talk by numerous examples.
Conclusions: A mesh may be a natural set up for a surface (c.f. subdivision methods). The anisotropic energy method gives rise to shapes that follow the mesh. It is well conditioned vs. the mesh.

4 Nef Polyhedra and Selective Geometric Complexes

Hanspeter Bieri

Nef Polyhedra were introduced by W. Nef in 1978, Selective Geometric Complexes (SGC) by J.R. Rossignac and M.A. O'Connor in 1990. Both notions refer to certain families of pointsets in d-space together with a number of appropriate "elementary" operations. It is shown that Nef Polyhedra can be understood as a special type of SGC, with important additional properties and specific representations. On the other hand, the SGC-model indicates a way to liberate Nef polyhedra from linearity without losing too many of their nice properties. The result is a new type of "solid" that should prove useful in dimension-independent geometric modeling.

5 Multidimensional Geometric Modelling and Variational Design

Adrian Bowyer, University of Bath

This presentation will describe how a multidimensional CSG geometric modeller that has polynomial and transcendental inequalities and hypersurfaces as primitives can be used to represent both the geometry and the behaviour of mechanical components simultaneously.
In particular, the concept of the omnimodel will be explained. This is a single hypermodel representing both three-dimensional geometry and changes that may happen to it as its parameters vary. Such parameters may describe solid-body transformations of objects, in which case together they form their configuration space, or changes in shape.
Configuration-space maps can be produced using these techniques, and explicit constraints can be intersected with them. This allows the behaviour of mechanisms to be modelled and displayed using animation. If, in addition, variations in key shape parameters are added as extra dimensions, satisfactory combinations of these parameters that produce the desired mechanism may be found before any physical prototyping is done at all.

6 A Declarative Modelling System

P. Michalik, Beat Brüderlin, TU Ilmenau

We investigate methods of using constraint-based modeling in a free-form curve and surface enviroment. In this work we concentrate on a problem of maintaining the curve-surface incidence relation while the curve is edited.
We formulate the relation between the degrees of freedom (DOF) and parameters (control points of the surface and curve resp.) as an explicit functional prescription without using the usual variational model.
We show how the polynomial composition algorithm based on blossoming and a data structures for efficient storage of intermediate results, significantly speeds up the computation and contribute to the robustness of the algorithm.
We sketch a solution to the known problem of the incompatibility of a general curve on a surface with the surface parametrization by locally changing the

parametrization of the original surface which makes a general domain curve to an iso-line in either parameter direction. We avoid the use of variational methods.

7 Direct Segmentation of Unorganized Point Sets Based on Nearest Neighbor Criteria

G. Brunnett, M. Vanco, T. Schreiber Universität Kaiserslautern

In reverse engineering one considers the problem of creating a CAD model for an already existing object. The reverse engineering process begins with the data acquisition phase where points are sampled from the surface of the object with a measuring device. The segmentation phase is concerned with structuring these points into segments such that the segments are appropriate for surface fitting.
In this talk we present an approach towards an automatic segmentation procedure that is direct in the sense that it operates directly on the point set and avoids the construction of a global triangulation. The method consists of three basic steps. First, the neighborhood graph of the point set is computed. Then, higher order surface information is derived on the basis of the neighborhood graph. Finally, the segmentation is performed based on this information. For all three steps we present different strategies and compare the efficiency of the algorithms and the quality of the obtained results.

8 Light-Field Modelling

Emilio Camahort, University of Texas at Austin

Image-based modelling and rendering has recently received much attention in the computer graphics community. Image-based models rely on the concept of light field, a 5D function that represents the radiance passing through a point in space in all possible directions. Early models represent the support of the light field using non-uniform parameterizations, that introduce geometric biases in the representation. Such biases produce noticeable artifacts at rendering time.
In this talk we study different light-field parameterizations. We advocate for two

uniform parameterizations: the two-points-on-a-sphere parameteriza- tion and the direction-and-point parameterization. We then compare them in terms of geometric errors produced after discretization. Finally, we con- clude that the direction-and-point parameterization introduces the least geometric errors while providing a uniform parameterization of the support of the light field.

9 Rational Bisectors of Rational Varieties in R^3

Gershon Elber, Technion - Haifa

We present a summary of some recent results on rational bisectors between two varieties in R^2 and R^3 and briefly present a constructive approach at the computation of the rational bisectors of curve-curve and point-surface in R^3. The generalization and existence of rational bisectors in higher dimensions is also considered.
In addition, we introduce and examine a new shape formulation we denote the alpha-sector. The alpha-sector extends the notion of the bisector to arbitrary distance ratios between the two varieties. We show that a close formulation, we denote the pseudo-alpha-sector, is indeed rational and can be useful to a whole variety of applications, including metamorphosis.

10 A Permanence Principle for Shape Control

Gerald Farin, Arizona State Univ.
Dianne Hansford, NURBS Depot

We analyze the discrete version of the bilinearly blended Coons patch which creates a rectangular control net from four input boundary polygons. We note two properties: a) the discrete Coons patch creates subquadrilaterals that are as close as possible (least squares sense) to parallelograms. b) consider a sub-control net. It has four control boundary polygons. If we apply the Coons patch to it, we create a control net which is identical to the one obtained from the original boundaries.
applying b) (the permanence principle) to all 3x3 subnets, we see that each inte-

rior control point is a certain linear combinations of its eight neighbors. Writing out all these relationships (masks), we have a linear system the solution of which is the discrete Coons patch. By varying the coefficients of the masks we obtain significant improvements over the shape of the discrete Coons patch.
We then apply the same principle to Bezier triangular control nets, surface fairing, and to mesh optimizaton of irregular triangular meshes.
We also compare our discrete Coons approach to variational approaches.

11 Wavelets on Triangulations

Michael S. Floater, SINTEF - Oslo

This talk studies algorithms for decomposition, reconstruction, and approximation based on piecewise linear prewavelets on bounded triangulations of arbitrary topology. The work has been carried out jointly with Ewald Quak and to some extent Martin Reimers, both colleagues at SINTEF. Our key mathematical result is showing that the Schur complement of the associated two scale matrix is symmetric, positive definite, and well-conditioned. Numerical examples suggest that thresholding based on prewavelets yields a smaller approximation error than when based on the simple ‘Faber’ decomposition scheme.

12 Blossoming, the Cancellation Axiom, and Divided Difference

Ron Goldman, Rice University

In this presentation, the blossoming axioms for polynomials are extended to include additional parameters along with a cancellation axiom, further unifying the theories of the multiaffine and multirational blossoms. It is then established that the divided difference operator can be completely characterized by a similar set of axioms. Thus the divided differences is shown to be intimately related to blossoming, and formulas are presented that express the divided differences of polynomials in terms of the multiaffine blossom. Several parallel identities are derived that exhibit this close connection between the two distinct forms of the blossom and the divided difference.

13 Applications of Contouring

Thomas Grandine

In the early 1990's, Boeing developed an accurate, robust, numerical contouring code for solving surface intersection problems. Since then, many additional and unexpected uses for that code have been found. This talk reviews the essentials of the numerical method, and outlines how it has been used to solve surface intersection and curve projection problems, to generate horizon lines on surfaces, and to create fillet surfaces and envelope curves, as well as additional applications that have arisen at Boeing in manufacturing and machining. hining.

14 A quintic G^1 triangular interpolating spline

Stefanie Hahmann, LMC-IMAG, Grenoble France
Georges-Pierre Bonneau, LMC-IMAG, Grenoble France

A piecewise quintic G^1 spline surface interpolating the vertices and optionally given normal vectors of a triangular surface mesh of arbitrary topological type is presented. The surface has an explicit Bézier representation, is affine invariant and has local support. The twist compatibility problem which arises when joining an even number of polynomial patches G^1 continuously around a common vertex is solved by constructing C^2-consistent boundary curves. Piecewise C^1 boundary curves and a regular 4-split of the domain triangle make shape parameters available for controlling the boundary curves. A small number of free inner control points can be chosen for some additional shape effects.

15 Integral spline surfaces with rational offsets

Bert Jüttler and M.L. Sampoli, TU Darmstadt

We present a construction for polynomial spline surfaces with a piecewise linear field of normal vectors. As main advantageous feature these surfaces possess exact rational offsets. The spline surface is composed of quartic Clough-Tocher-type macro elements. Each element is capable of matching boundary data consisting of three points with associated normal vectors. The collection of the macro elements forms a G1 continuous spline surface. With the help of a reparamaterization technique we obtain an exact rational representation of the offset surfaces by rational triangular spline surfaces of degree 10.

16 C-continous Shape-Preserving Interpolants of Variable Degree

Panagiotis Kaklis, National Technical University of Athens

In this paper we introduce a new family of C^4-continuous variable-degree polynomial splines. We consider the asymptotic properties of these splines, as the degrees increase, locally, semi-locally and globally, and study the structure of their Bézier control polygon. We then develop an automatic global iterative algorithm for constructing C^4 interpolatory curves, which match the convexity and torsion information implied by the polygonal interpolant. Finally, the performance of the algorithm is tested for a number of planar and spatial data sets.

17 Topological Techniques in Intersecting Surfaces of Special Types

Myung-Soo Kim, Seoul National University

This talk introduces topological techniques for computing the intersection curve

of two surfaces of special types.
The torus is an envelope surface of a sphere moving along a circle; thus the intersection between a torus and another surface can be understood as an envelope curve of a one-parameter family of sphere-surface intersection curves. Based on this interpretation, we develop simple algorithms for intersecting a torus with simple surfaces such as natural quadrics or another torus.
Given a sphere and a surface of revolution, consider the plane determined by the center of the sphere and the rotation axis of the surface of revolution. This plane cuts the given sphere in a great circle and the surface of revolution in the generating curve. The topological type of the surface intersection curve is completely determined by the planar arrangement of the great circle, its symmetry circle, and the generating curve. The problem is thus essentially reduced to that of intersecting a planar curve with two circles.
Finally, we consider the intersection of two ruled surfaces. Each ruled surface is linear in its ruling direction. Two linear parameters can be easily eliminated from three equations representing the x, y, z-coordinates for the intersection of two ruled surfaces. Thus we get one implicit equation in two remaining parameters, which essentially reduces the surface intersection problem to the construction of an implicit curve.

18 Voronoi Diagrams and Strategies for Autonomous Robots

Rolf Klein, FernUniversität Hagen

The first part of the talk contains a survey on ongoing research on Voronoi diagrams. We mention the crust algorithm by Amneta, Bern, and Eppstein for reconstructing a curve from sufficiently dense sample points. Then, variations of the distance measure, the type of sites, and the dimension are discussed, that underlie the construction of the Voronoi diagram. Abstract Voronoi diagrams are introduced, as a unifying concept for the planar case. We demonstrate that in 3D there can be an unbounded number of homothetic strictly convex, smooth spheres that contain 4 given points on their surfaces. Moreover, we mention Tagansky's upper bound for the complexity of the resulting Voronoi diagrams. In the second part of the talk we investigate how a point-shaped mobile robot equipped with a 360 degree vision system can efficiently explore an unknown simple polygon. For analyzing the resulting paths, the angle hull of a set is introduced; its boundary can be at most twice as long as the perimeter of the set. This leadss to a

27-competitive strategy for the polygon exploration problem.

19 Multiresolution Modeling on Meshes

Leif Kobbelt, Max-Planck-Institut für Informatik - Saarbrücken

The use of polygonal meshes for the representation of highly complex geometric objects has become the de facto standard in most computer graphics applications. Especially triangle meshes are preferred due to their algorithmic simplicity, numerical robustness, and efficient display. The possibility to decompose a given triangle mesh into a hierarchy of differently detailed approximations enables sophisticated modeling operations like the modification of the global shape under preservation of the detail features.
So far, multiresolution hierarchies have been proposed mainly for meshes with subdivision connectivity. This type of connectivity results from iteratively applying a uniform split operator to an initially given coarse base mesh. In this talk I present techniques to build hierarchical structures for arbitrary meshes with no restrictions on the connectivity. Since smooth (subdivision) basis functions are no longer available in this generalized context, I use constrained energy minimization to associate *smooth* geometry with *coarse* levels of detail.

20 Triangulating 3D Surfaces from Unorganized Points

Geza Kos, Hungarian Acadamy of Science

Reverse engineering is the process of converting measured data points of an existing object into a CAD model. The data points are typically measured by 3D laser scanners from multiple views. These point clouds need to be united, thus finally an unorganized (scattered) 3D point set is obtained. For further processing, the data points are generally arranged into a triangular structure, which provides adjacency and connectivity information for further processing steps.
While triangulating a point set over a plane of projection is relatively straightfor-

ward, to construct a consistent 3D triangulation is a difficult problem in general. Previous solutions - see for example [1], - were found quite restricted for certain types of data sets and computationally inefficient in many cases.
The present talk addresses the problem of 3D triangulation with special emphasis on generality, uneven point distributions with holes and computational efficiency. The algorithm is based on the generalization of Delaunay tessellation of 2D manifolds. After preprocessing steps, such as clustering and normal estimation, local triangulations are built around each vertex. These triangulations are merged together step by step until a complete and consistent triangular mesh is obtained. Filling holes and a smoothing phase terminate the procedure. Several issues of computational efficiency are also discussed and a few complex test examples are shown.

References

[1]
H. Hoppe et al. *Surface Reconstruction from Unorganized Points.*
Computer Graphics, Vol. 26 (1992), No. 2, pp. 71-76
[2]
H. Edelsbrunner, E. P. Mücke. *Three-Dimensional Alpha Shapes.*
ACM Transaction on Graphics, Vol. 13 (1994), No. 1, pp. 43-72

21 Simulated Annealing and the Construction of Linear Spline Approximations to Scattered Data

Oliver Kreylos

We describe a method to create optimal linear spline approximations to arbitrary functions, given as scattered data without known connectivity. We start with an initial approximation consisting of a user-supplied number of vertices and improve this approximation by choosing different vertices, governed by a simulated annealing algorithm. In the case of one variable, the approximation is defined by line segments; in the case of two variables, the vertices are connected to form a Delaunay triangulation of the selected subset of sites in the plane. In a second version of this algorithm, specifically designed for the bivariate case, we choose vertex sets and also change the triangulation to achieve both optimal vertex placement and optimal triangulation. We then create a hierarchy of linear spline approximations, each one being a superset of all lower-resolution ones.

22 Theory and algorithms for non-uniform spline wavelets

Tom Lyche, Univ. of Oslo

Wavelet decomposition and reconstruction is a useful tool in geometric modelling. We consider spline wavelts of minimal support for decomposition of a given B-spline curve or surface. To define these wavelets suppose S_0 and S_1 are univariate B-spline spaces with $S_0 \subset S_1$. Let W_0 be the orthogonal complement of S_0 in S_1 with respect to the usual L^2 inner product. Any member w of W_0 can be written as a linear combination of the B-splines in S_1. The set of indices used for this representation is called the index support of w. A basis of W_0 with each member having minimal index support is called a basis of B-wavelets. For general univariate splines B-wavelets were studied by Mørken and the author in a paper in 1993 and a simple algorithm for determining the index support of these functions were given. In this talk we show that a basis of B-wavelets is unique (apart from scaling). We also discuss fast and stable algorithms for decomposition and reconstruction using B-wavelets. The talk is based on joint work with Knut Mørken and Ewald Quak.

23 Cylindrical Surface Pasting

Stephen Mann, University of Waterloo

The idea of hierarchical modeling is that many surfaces have different levels of detail and for modeling purposes it is useful to interactively edit these surfaces at any level of detail. Detail can be added to tensor-product B-splines via knot insertion, but once the detail has been added the the surface can not be edited at lower level of detail. Hierarchical B-splines were designed to allow for a type of knot insertion method that still admits editing the surface at any level of detail.

However, the details can not be rotated, nor is it convenient to create a library of such details. Surface pasting is a generalization of hierarchical B-splines that allows for the creation of a feature library, and where the pasted features can be rotated and scaled, and translated across the base surface.
Cylindrical pasting is a parametric-blending method that creates a smooth transition surface between a pair of B-spline surfaces that do not originally intersect. This blending surface is a deformed cylinder, and its creation is based on the surface pasting composition method, which adds detailed features to base surfaces by means of an efficient displacement method. In cylindrical pasting, a transition cylinder can be pasted on a NURBS surface or onto a NURBS cylinder. A displacement scheme is used to locate the control points of the blending cylinder to achieve approximate C^1 continuity between the boundaries of the base surfaces and the edges of the cylinders.

24 Low-Discrepancy Sequences for Computing Volume Properties

Ralph Martin, University of Wales

This talk investigates the use of low-discrepancy sequences for computing volume integrals in geometric modelling. An introduction to low-discrepancy point sequences is presented which explains how they can be used to replace random points in Monte Carlo methods. The relative advantages of using low-discrepancy methods compared to random point sequences are discussed theoretically, and then practical results are given for a series of test objects which clearly demonstrate the superiority of the low-discrepancy method.

25 An affine invariant norm based upon the Loewner ellipsoid

Volker Milbrandt, Universität Stuttgart

Motivated by the work of Nielson (1987), an affine invariant norm based upon

the Loewner ellipsoid will be constructed. This norm has the advantage of an obvious geometric meaning. Affine invariant norms are always dependent from the set V of given data points. Therefore, in general the norm changes if a further point P is added to the set V. However, the norm based on the Loewner ellipsoid will be preserved unchanged as long as the additional point P is located in the interior of the Loewner ellipsoid of the originally given point set V.
A combinatorical deduction of the gauge ellipsoid is given provided the interpolating minimal ellipsoids are known for each subset of exactely $d+1$ points in R^d. Further, the Loewner ellipsoid is constructed inductively for this special case by some geometric observations.
The talk concludes by giving some numerical comparisons of the two affine invariant norms using results of scattered data interpolation.

References

[1]
Gregory M. Nielson
Coordinate free scattered data interpolation
In C. K. Chui, L. L. Schumaker, and F. I. Utreras, editors
Topics in Multivariate Approximation, pages 175–184, Boston, 1987. Academic Press.

26 Implicit Surface Approximation Using Adaptive Spatial Meshes

Heinrich Müller, Universität Dortmund

An algorithm is presented which calculates surface meshes approximating an implicitly defined surface. The algorithm first performs an adaptive spatial tetrahedrization from which the surface is extracted by a marching tetrahedrons approach afterwards. One improvement over other known implementations of this approach is the context-free calculation of the spatial tetrahedrization which is more efficient than existing context-sensitive or two-pass solutions. Furthermore, an improved quality of the surface mesh is achieved by setting the subdivision points on the surface so that the resulting surface mesh is part of the resulting spatial mesh.

27 Discrete Algorithms for Qudrilateral and Hexahedral Mesh Generation

Matthias Müller-Hannemann, Technische Universität Berlin

We survey new algorithms for quadrilateral surface and hexahedral volume mesh generation by means of combinatorial optimization and graph algorithms.
In the surface mesh refinement problem we are given a coarse mesh of polygons in three-dimensional space. The problem is to find a decomposition into strictly convex quadrilaterals such that the resulting mesh is conforming and satisfies prescribed local density constraints. We present the ideas of our new approach based on network flow techniques. For meshes with branchings we introduce a two-stage approach which first decomposes the mesh into branching-free components, guarantees feasibility by solving a certain system of linear equations over GF(2), and finally uses minimum cost bidirected flows on the components.
In the second part of the talk, we sketch a new method for constructing all-hexahedral meshes. The core of our method is to build up a compatible combinatorial cell complex of hexahedra for a solid body which is topologically a ball and for which a quadrilateral surface mesh of a certain structure is prescribed. The step-wise creation of the hex complex is guided by the cycle structure of the combinatorial dual of the surface mesh. Our method transforms the graph of the surface mesh iteratively by changing the dual cycle structure until we get the surface mesh of a single hexahedron. Starting with a single hexahedron and reversing the order of the graph transformations, each transformation step can be interpreted as adding one or more hexahedra to the so far created hex complex.

28 Exploiting Wavelet Coefficients for Modifying Functions

Alexa Nawotki, Universität Kaiserslautern

Cryptographical algorithms usually encode all the information stored in the original data, although very often only a few details are secret and must be hidden. I present a method that computes intermediate versions of the data with gradually reduced information. For that purpose I use a wavelet-decomposition and look for connections between the decomposition-level and the included information.

As an application of this method the reflector of a headlight is investigated: The reflection pattern which is produced by the reflector is modified selectively while the geometrical form is (almost) preserved.
This approach was tested with Haar-wavelets and with B-spline-wavelets. In the first case the function must be described with hat-functions as base functions, but it is easy to avoid changes in an arbitrary region, and the error can be estimated a priori. Error estimates are more difficult in the second case, but B-splines are more appropriate in the context of CAGD.

29 Control net approximation to splines

Jörg Peters, University of Florida
joint work with David Lutterkort

The distance between a spline $s = bN = \sum_{i=0}^{d} b_i N_i^d$ with Greville abscissae t_i^* and its control polygon ℓ on the interval $[t_k^*, t_{k+1}^*]$ is expressed in the nonnegative, convex basis functions β_{ki} and second differences $\Delta_2 b_i$ as

$$s - \ell = (\Delta_2 b)\beta_k = \sum_{i=0}^{d} (\Delta_2 b_i)\beta_{ki}.$$

Application of the L^r norm, $|\cdot|_r$, and Hölder's inequality for vectors with $1/p + 1/q = 1$ yields the families of bounds

$$|s - \ell|_r \leq \|(\Delta_2 b)||_p |\|\beta_k\|_q|_r.$$

Tighter bounds follow by separating $(\Delta_2 b)$ into the vector of nonnegative second differences $(\Delta_2^+ b)$ with $\Delta_2^+ b_i = \max\{\Delta_2 b_i, 0\}$ and the vector of nonpositive second differences $(\Delta_2^- b)$:

$$(\Delta_2^- b)\beta_k \leq s - \ell \leq (\Delta_2^+ b)\beta_k.$$

In particular, since $\ell + (\Delta_2^+ b)\beta_k$ is convex, it can be bounded above by $\overline{e}$, its linear interpolant at the Greville abscissae t_k^* and t_{k+1}^*, and below by the analogous $\underline{e}$. This yields a piecewise linear *envelope* consisting of

$$\underline{e} \leq s \leq \overline{e}$$

on the interval $[t_k^*, t_{k+1}^*]$. Such envelopes are the basis for reducing a continuous nonlinear feasibility problem of 1-sided and 2-sided spline approximation to a linear program. Examples illustrate the effectiveness of this approach for computing smooth spline paths that stay within a given polygonal channel.

30 Error Propagation in Geometric Constructions

Helmut Pottmann, TU Wien

Assuming the presence of errors in the input of a geometric construction, we study the effect of the input errors on the output. At first, we deal with error propagation problems of the following type: We are given a set of points, and a point dependent on them. The initial points are allowed to vary independently in some domains. The question is to describe the possible loci of the final point. We analyse this problem for several types of dependence and several types of domain where the original points are allowed to move in. One of the most simple cases is that of affine or linear dependence, and of convex domains. It is tied to the concept of Minkowski sums from convex geometry. This has numerous applications to spline curves and surfaces. It is also shown that a careful study of simple metric constructions, such as a circle through three points, becomes quite involved in the presence of input errors.

31 A Shape Modelling API for the STEP Standard

Michael J. Pratt, RPI/NIST, USA

The international standard ISO 10303 (STEP) is intended for the transfer of computer aided design (CAD) and other related types of electronic product models between different application systems. The first release of STEP (dated 1994) can transfer shape models of the boundary representation and similar types, formulated in terms of explicit geometrical and topological elements. Modern CAD systems generate additional information, concerning parametrization, constraints and form features, which currently gets lost during STEP model transfers because the standard provides no means for exchanging it. The value of the lost information lies in its control of the behaviour of the model under modification. It embodies important aspects of 'design intent', and without it there is no guarantee that editing operations performed after the transfer of a model into a receiving system will be valid in the sense of preserving the functionality of the designed object. The loss of this behavioural information in a model transfer leads to

the expenditure of extensive operator time in efforts to reconstruct the missing design intent, often through a process of trial and error. Consequently, an effort is now under way to extend the STEP standard to include parametrization and constraint data, with a further extension into the form features area to follow shortly.
Two approaches are being taken. The first is to supplement the current explicit type of model transfer by constraint and parametrization data. The second is to transfer 'procedural' or 'history-based' models whose shapes are defined implicitly in terms of the sequence of constructional operations used to create them. Many current CAD systems use a combination of these two approaches for their master model representations, and so it is intended that the two different kinds of model will be mutually compatible within the standard. The presentation given will concentrate on problems and progress in the transfer of procedurally defined shape models. It has been determined that this will require the development of a standardized Applications Programming Interface (API) for CAD systems.

32 Convexity Preserving Interpolation

Hartmut Prautzsch, Universität Karlsruhe

A geometric criterion is derived for the convexity of certain simple Powell-Sabin interpolants with well-associated tangent planes.
If this criterion is not satisfied or if the tangent planes are not well-associated but stem from a convex function then there still exists a finer and convex Powell-Sabin interpolant with well-associated tangent planes. Although its existence is proven it is open how to construct such a finer interpolant.
There are also finer convex Powell-Sabin interpolants without well-associated tangent planes. Again it is not known how to construct them and their construction appears more difficult since the convexity criterion is more complicated than the one presented in this talk.

33 Best Bounds on the Approximation of Splines by their control net

Ulrich Reif, Universität Stuttgart

We present best bounds on the deviation between univariate polynomials, Bézier triangles, and splines and the corresponding control structure. The analysis is based only upon three elementary principles, namely

- Positivity of the basis.
- Linear Precision.
- Positivity of second differences of the control points implies convexity of the control polygon.

¿From best bounds for the difference at a fixed argument we derive best bounds on arbitrary L_p-norms, $1 \leq p \leq \infty$.
For instance we obtain the following result on a spline $B^n P$ of degree n with knot sequence T and control point sequence P: Let $\Delta^2 P$ be the coefficients of the second derivative, i.e. $D^2 B^n P = B^{n-2}\Delta^2 P$, and denote the variance of always d consecutive knots by

$$\sigma_j^2 := \frac{1}{n-1} \sum_{i=j+1}^{j+d} T_i \, ,$$

then the difference between the spline $B^n P$ and its control polygon $H^n P$ is bounded by

$$\|B^n P - H^n P\|_\infty \leq \frac{\max_j \sigma_j^2}{2n} \|\Delta^2 P\|_\infty \, .$$

Equality holds if $B^n P$ is a quadratic polynomial.

34 Recent Advances in Macro-Element Methods for Fitting Scattered Data

Larry L. Schumaker, Vanderbilt University

Many of the most successful methods for fitting surfaces to bivariate scattered data are based on piecewise polynomials defined over either a rectangular or triangular partition of the domain of interest. These types of methods are particularly effective when the interpolation process can be described *locally*, i.e., the polynomial piece of the spline associated with a given subset T can be constructed from data associated with points in (or near) T. Such methods are often referred to as *macro element methods*. Among the best-known are the classical polynomial, Clough-Tocher, and Powell-Sabin elements. In this presentation we show how recent results in the theory of splines over triangulations can be used to construct new macro elements and improve existing ones in terms of smoothness, accuracy, degrees of freedom, and amount of derivative data required.

35 Implicit Surfaces Revisited - the I-patches

Tamas Varady, Pal Benko, Geza Kos and Alyn Rockwood(*)
Hungarian Academy of Sciences and (*) Arizona State University

Generating smooth, connecting surfaces between given primary surfaces is one of the central problems of Computer Aided Geometric Design. While several parametric surface representations are known for defining n-sided patches, implicit techniques have been used only under strict limitations. This was due to problems experienced while using various composition methods, which resulted in high degree polynomials with self-intersections, branching and uncontrollable shapes. The purpose of the current paper is to bring back implicit formulations to light and define complex n-sided surfaces by improving and extending former methods.
Using the so-called I-patch formulation, it possible to obtain user controllable shapes connecting smoothly arbitrary number of primary surfaces. The primary surfaces are typically simple implicit surfaces, however, the method can be generalised for parametric surfaces as well. The boundaries of the patch are given as intersection curves between the primary surfaces and associated bounding surfaces. The degree of geometric continuity is an important parameter of the formula, which defines higher order smoothness in a straightforward way. The shapes can be controlled by individual weights assigned to the primaries. It is also possible to set a middle point for the interior of the shape. Variations on how to compute distance measures of the component surfaces - algebraic, normalised and Euclidean methods - are also discussed and analysed. Several examples are given to illustrate the feasibility of the approach.

36 Subdivision Schemes for Flow

Henrik Weimer and Joe Warren, Rice University

The motion of fluids has been a topic of study for hundreds of years. In its most general setting, fluid flow is governed by a system of non-linear partial differential equations known as the Navier-Stokes equations. However, in several important settings, these equations degenerate into simpler systems of linear partial differential equations. This paper will show that flows corresponding to these linear equations can be modeled using subdivision schemes for vector fields. Given an initial, coarse vector field, these schemes generate an increasingly dense sequence of vector fields. The limit of this sequence is a continuous vector field defining a flow that follows the initial vector field. The beauty of this approach is that realistic flows can now be modeled and manipulated in real time using their associated subdivision schemes.

37 Shape Parametrisation for Biomedical Applications

M.I.G. Bloor and M.J. Wilson, University of Leeds

The aim of this research is to develop a method for the parametric description of the geometry of biological objects with complex shapes, so that realistic analysis of their functioning and development can be carried out. This paper illustrates the work through descriptions of two areas: the shape parameterisation of the human heart (where the complex changes in shape that take place during a heart beat need to be modelled in order to understand the fluid mechanics), and the parameterisation of the shapes adopted by lipid vesicles (which are often used as models for the shapes adopted by biological membranes). In both these ecaes a realistic representation of geometry and shape is required if the behaviour of the actual system is to be understood and the accuracy of the underlying biophysical models confirmed.

38 Analytical Concepts and Tools for Computing Geodesic Voronoi Diagrams and Geodesic Medial Axes on Free Form Surfaces

Franz-Erich Wolter, Universität Hannover

The Voronoi Diagram of a finite point set P in the Euclidean plane E is given by the boundaries of all proximity regions with respect to all single points in the given set P. The proximity region $V(p, P)$ of a point p with respect to the set P is given by the points in the plane E that are in their Euclidean distance closer to p than to all other points in P. The medial axis of a bounded domain D in E whose boundary is given by a piecewise curvature continous curve consists of all centers of all discs of maximal size that are still contained in D. It has been shown by the author that the Medial Axis of the domain D is the closure of all those points in D that have at least two distance minimal segments to the boundary set of D. Voronoi diagram and Medial axis are fundamental objects of computational geometry. It is a very natural problem to compute both objects in the more general situation where the euclidean plane E is replaced by a free form surface and where the distance between two points is now defined by the geodesic distance on the surface given by the length of a shortest surface curve joining the two points. We explain how tools and concepts from Riemannian geometry such as Jacobi-fields and geodesic focal curves can be used efficiently to obtain highly accurate computations of geodesic Voronoi diagrams and of the geodesic medial axis on parametric surfaces. We also show examples where the presented tools have been applied to compute geodesic Voronoi Diagrams and to compute geodesic Medial Axes for simply connected domains on real analytic parametric surfaces. We assume for the latter examples that the boundary of the simply connected domain D is given by a piecewise real analytic boundary curve and we show that in this case the Medial axis must be topologically a finite tree being a deformation retract of D. The (end) vertices of this tree are located on the geodesic focal curve $F(t)$ of the boundary curve $B(t)$ of D. At those (end) vertices (of the Medial Axis tree) the geodesic distance of the focal curve $F(t)$ to the boundary curve B has local minima.

39 Applications of Perturbation Theory to B-Spline Interpolation and Approximation

Hans Wolters, Hewlett Packard Labs - Palo Alto

This talk illustrates that in many practical applications it is essential to have an estimate on the condition of the interpolation or approximation operator. This is especially true for fitting of noisy data. We develop some bounds on the norm of B-Spline interpolation and approximation operators by means of matrix analysis. We introduce the theory of H-Matrices and show how these can be used to get estimates more efficiently. The second part of the talk shows how the results obtained so far lead to a heuristic for chosing knots to a given parameterization. We show how knots might be obtained that minimize the sensitivity to perturbations in the input data.