Quantum, concretely, abstractly

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(Comprises joint work with Samson Abramsky)

What does "quantum" stand for? For us it stands for the concepts (both operational and formal) which had to be added to classical physics in order to understand observed phenomena such as the structure of the spectral lines in atomic spectra, experiments exposing non-local correlations, seemingly 4π symmetries etc. While the basic part of classical mechanics deals with the (essentially) reversible unitary dynamics of physical systems, quantum required adding the notions of measurement and (possibly non-local) correlations to the discussion. The corresponding mathematical formalism was considered to have reached maturity in [3], but there are some manifest problems with that formalism:

(i) While measurements are applied to physical systems, application of their formal counterpart (i.e. a self-adjoint operator) to the vector representing that state of the system in no way reflects how the state changes during the act of measurement. Analogously, the composite of two self-adjoint operators has no physical significance while physically, measurements can be effectuated sequentially. More generally, the formal types in von Neumann's formalism do not reflect the nature of the corresponding underlying concept.

(ii) Part of the problem regarding the measurements discussed above is that in the von Neumann formalism there is no place for storage, manipulation and exchange of the classical data obtained from measurements. Protocols such as teleportation involving these cannot be given a full formal description.

(iii) The behavioral properties of entanglement which for example enable continuous data exchange using only finitary communication are hidden in the formalism. Only in [2] they were exposed, but still not well understood.

The recent work in [1] addresses all these problems, and in addition provides a purely categorical axiomatization of *the abstract quantum*. The concepts of the abstract quantum are formulated relative to a strongly compact closed category with biproducts (of which the category **FdHilb** of finite dimensional Hilbert spaces and linear maps is example). Preparations, measurements, either destructive or not, classical data exchange are all morphisms in that category, and their types fully reflect the kind. Correctness properties of standard quantum protocols can be abstractly proven, and in this seemingly purely qualitative setting even the quantitative Born rule arises. Taking **FdHilb** as this category provides *the concrete quantum*, in which, of course, all the above problems are addressed. In particular, the properties exposed in [2] are perfectly captured by the compact closure of **FdHilb**.

We intend to address the following remaining issues:

- Mixed states, mixed data transformations and mixed measurements involve introduction of cognitive uncertainty besides the probabilities which arise due to quantum measurements. Hence the abstract quantum should be extended with a cognitive layer to analyze protocols which involve 'forgetting', 'information loss', 'decoherence' etc.
- The role of space is crucial in certain quantum protocols since classical information exchange is restricted to the light cones. Analysis of such protocols hence requires blending quantum with causal structure, and also with a space-time metric if one intends qualitative analysis.

These features will allow analysis of many different kinds of protocols (in particular, other than the ones addressed in [1]) and to ask and answer questions which could not even be rigorously formulated in the von Neumann formalism.

References

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