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Minorant methods for stochastic global optimization

Keywords: Stochastic global optimization, stochastic tangent minorant, branch and bound method.

Branch and bound method and Pijavskii's method are extended for solution of global stochastic optimization problems. These extensions employ a concept of stochastic tangent minorants and majorants of the integrand function as a source of global information on the objective function. A calculus of stochastic tangent minorants is developed.

Problem of stochastic optimization consists in minimization of a mathematical expectation or a probability function. Difficulty of the problem is that the objective function cannot be calculated precisely, but only statistical estimators of its values and, probably, of its gradients are known. The task is in the search of local and global minima of the problem with the use of these estimators. Extensive literature is devoted to the solution of convex stochastic optimization problems. Problems and methods of searching local minima in nonconvex stochastic optimization problems are discussed in [1]. A number of global stochastic optimization problems and a stochastic branch and bound method for their solution are studied in [9 - 11], where estimations of branches (subtasks) are obtained by means of the so-called interchange relaxation, i.e. by interchange of minimization and integration (mathematical expectation or probability) operators. In works [4, 5, 8] the specified stochastic branch and bound method is applied to global optimization of probabilities with application to the control of environmental contamination, and in [2, 3] it is applied to optimal routing and project management problems.

The basic results of the present work are developed in [6, 7, 12 - 16] and consist in extension of deterministic Pijavski's global optimization method and a classical branch and bound method on problems of global stochastic optimization (with mathematical expectation and probability objective functions).

Piyavskii's method has been repeatedly rediscovered and is one of popular methods of deterministic global optimization. It has two equivalent forms: for optimization of maximums functions and for functions admiting the so-called tangent minorants [6]. The concept of tangent minorants is the key one for the given method. The basic problem of Piyavskii's method in a multivariate case is how to solve auxiliary approximating multiextremal problems.

A common feature of considered methods is the use of tangent minorants of the objective function as a source of global information on the function behavior. Thus the key problem is how to construct tangent minorants. For example, as such minorants tangent cones or tangents paraboloids to the function graph can be used. The use tangent paraboloids instead of cones considerably increases efficiency of the method [12]. In [6] a calculus of tangent minorants for

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complex nonconvex criterion functions is developed. In the present paper we give new rules for calculation of tangent minorants, in particular, for minimum and maximum functions and also stochastic tangent minorants for mathematical expectation and probability functions. Consideration of stochastic minorants is similar to generalization of a deterministic gradient method to a stochastic quasigradient method for solution of stochastic programming problems. The search of deterministic minorants as well as gradients of mathematical expectation functions can be problematic, however calculation and use of stochastic minorants and stochastic quasigradients is quite possible.

A common feature of considered methods applied to a problem of global stochastic optimization is the use of a sequence of uniform approximations of the objective function and tangent minorants of these approximations. Thus, we obtain new modifications of Piyavskii's method and of the branch and bound method for the solution of so-called limit extremal problems in which objective function is optimized through a sequence of approximating functions.

We radically solve a problem of solution of auxiliary approximating problems in a multivariate Piyavskii's method, namely, we solve them not precisely (that is rather difficult) but approximately by partitioning the domain of search into subsimplexes and by searching a subsimplex with the least estimate from below of the approximating function (instead of searching points of its global minimum). Then this variant of the method, in essence, turns into a branch and bound method with minorant estimates of branches.

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