Tracking Mobile Users

Leah Epstein* Asaf Levin †

Cellular telephony systems, where locations of a mobile users may be unknown at some times, are becoming more common. Mobile users are roaming in a zone. A user reports its location only if it leaves the zone entirely. We consider cellular zones with n cells and m mobile users roaming among the cells. The location of the users is uncertain and is given by m probability distribution vectors. The Conference Call Search problem (CCS) deals with tracking a set of mobile users, in order to establish a call between all of them. The search is performed in a limited number of rounds, and the goal is to minimize the expected search cost. In the "unit cost model", a single query for a cell outputs a list of users located in that cell. The "bounded bandwidth" model allows a query for a single user per cell in each round. We discuss three types of protocols; oblivious, semi-adaptive and adaptive search protocols. An oblivious search protocol decides on all requests in advance, and stops only when all users are found. A semi-adaptive search protocol decides on all the requests in advance, but it stops searching for a user once it is found. An adaptive search protocol stops searching for a user once it has been found (and its search strategy may depend on the subsets of users that were found in each previous round). We establish the differences between the distinct protocol types and answer some open questions which were posed in previous work on the subject.

Polynomial time algorithms for finding optimal semi-adaptive and adaptive search protocols for two users

We describe one of our results. We consider the problem of computing an optimal semi-adaptive search protocol for tight instances of CCS. Below

*Department of Mathematics, University of Haifa, 31905 Haifa, Israel. lea@math.haifa.ac.il. Research supported by Israel Science Foundation (grant no. 250/01).

†Department of Statistics, The Hebrew University, Jerusalem, Israel. levinas@mscc.huji.ac.il.

Dagstuhl Seminar Proceedings 05031
Algorithms for Optimization with Incomplete Information
http://drops.dagstuhl.de/opus/volltexte/2005/58

we show a polynomial time algorithm for solving this problem for two users. We describe a fast algorithm to solve the two-users two rounds case (this solution holds for adaptive systems as well).

We assume that there are two users and two rounds and $B = 1$. Bar-Noy and Naor [2] showed that computing an optimal oblivious protocol for this case is an NP-hard problem. They left as an open question to decide if computing an optimal adaptive search protocol is polynomially solvable. We note that in this case the semi-adaptive search protocol is equivalent to the adaptive search protocol. Therefore, by computing an optimal semi-adaptive search protocol in polynomial time, we provide a positive answer for this question.

Our algorithm, denoted by *Alg*, guesses k that is defined as the number of cells that an optimal solution pages for the first user in the first round. This guess is implemented by an exhaustive enumeration using the fact that k is an integer in the interval $[0, n]$, and then returning the best solution obtained during the exhaustive enumeration. We next analyze the iteration in which the guess is correct.

Denote by $I_i^k = p_{1,i} \cdot (n-k) - p_{2,i} \cdot k$ the *index* of cell i in the k-th iteration. Our algorithm sorts the indices of the cells in non-decreasing order, and then it picks the first k cells (in the sorted list). These picked cells are paged for the first user in the first round, whereas the other cells are paged for the second user in the first round.

Theorem 1 *Alg returns an optimal semi-adaptive search protocol.*

The next corollary answers the open question raised by [2].

Corollary 2 *Alg returns an optimal adaptive search protocol.*

Open questions

We list several open questions that are left for future research:

- Determine the complexity status of computing an optimal adaptive search protocol for tight instances with $d = m > 2$.
- Find an FPTAS or prove its non-existence (by showing that the problem is NP-hard in the strong sense for fixed values of $d = m$) for computing an optimal oblivious search protocol for an arbitrary tight instance.

- Find a PTAS or prove its non-existence (by showing that the problem is APX-hard) for computing an optimal oblivious search protocol for an arbitrary tight instance. The running time of the PTAS should be polynomial in n and in $d = m$.

References

[1] A. Bar-Noy and G. Malewicz. Establishing wireless conference calls under delay constraints. *Journal of Algorithms*, 51(2):145–169, 2004.

[2] A. Bar-Noy and Z. Naor. Establishing a mobile conference call under delay and bandwidth constraints. In *The 23rd Conference of the IEEE Communications Society (INFOCOM2004)*, volume 1, pages 310– 318, 2004.

[3] L. Epstein and A. Levin. A PTAS for delay minimization in establishing wireless conference calls. In *Proc. of the 2nd Workshop on Approximation and Online Algorithms (WAOA2004)*, 2004. To appear.

[4] M. R. Garey and D. S. Johnson. *Computer and Intractability.* W. H. Freeman and Company, New York, 1979.

[5] D. Goodman, P. Krishnan, and B. Sugla. Minimizing queuing delays and number of messages in mobile phone location. *Mobile Networks and Applications*, 1(1):39–48, 1996.

[6] C. Rose and R. Yates. Minimizing the average cost of paging under delay constraints. *Wireless Networks*, 1(2):211–219, 1995.