Report of the Dagstuhl seminar on

Geometric Modelling

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organized by

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Geometric Modeling is the branch of Computer Science concerned with the efficient acquisition, representation, manipulation, reconstruction and analysis of 3-dimensional geometry on a computer. Models and shapes in 3-dimensions can be represented as splines or subdivision surfaces, as well as by polygonal meshes or point clouds. Applications of geometric modeling cover a wide collection of areas from classical computer aided design, reverse engineering and simulation, to computer graphics, scientific visualization, medical imaging, multimedia and entertainment.

The 6th Dagstuhl seminar on geometric modelling was attended by 59 participants. The participants came from 4 continents and 19 countries (!), and included 4 industrial scientists as well as the leading academic experts in the field. Several young invited researchers were funded by the HLSC program of the European community. A very special event during the conference was the award ceremony for the John Gregory Memorial award. This time Prof. Rida Farouki, Prof. Ron Goldman and Prof. Richard Riesenfeld have been awarded with this price for their fundamental contributions to the field of geometric modelling. After the conference, as with all previous Dagstuhl Seminars on Geometric Modelling, conference proceedings will be published in collaboration with Springer.

There were a total 53 technical presentations at the conference related to the following diverse topics:

- curve and surface modelling
- surface reconstruction
- surface interpolation and fitting
- multiresolution representations, subdivision surfaces
- algebraic methods for curves and surfaces
- 3D meshes
- computational topology
- geometric models for Biomedical application

Despite the very large number of presentations during the conference and the high attendance at these talks, there was time for scientific discussions and research.

The organizers would like to thank all the attendees for their participation. They would also especially like to thank the team of Schloss Dagstuhl for helping to make this workshop a success. As always, we enjoyed the warm atmosphere of the Schloss, which supports formal discussions along with informal exchanges of ideas.

List of talks

Abstracts

Geometric invariants of triangular quadric patches

Gudrun Albrecht
University of Valenciennes, France

We consider a rational triangular Bzier patch of degree 2. In general its underlying implicit surface is a so-called Steiner surface, an algebraic surface of order 4, which under certain conditions for the control points and the weights becomes a quadric surface. An easy method for determining, if the given patch lies on a quadric, and if so, for establishing the quadric's affine type, has been presented in CAGD 15(1998). On this basis we consider the problem of determining the directions of the principal axes of the underlying quadric of the given patch as well as the position of the origin yielding the quadric's normal form.

Non-conforming shape representations

Marc Alexa
TU Darmstadt, Germany

Non-conforming surface representations are based on a set of primitives scattered over the surface. These primitives could be points or low degree polynomial patches. We find this representation interesting, because it is often easier to generate from a given surface description and it might yield smaller Hausdorff error for the same number of primitives compared to continuous (i.e. connected) surface representations.

We discuss two aspects of non-conforming surface representations: First, how to generate them based on spatial subdivision and, second, how to render them efficiently without conversion to a mesh.

Splat representation of parametric surfaces

D. Ayala
Universitat Politècnica de Catalunya, Barcelona, Spain

Point-based geometry representations and their splat-based generalisations have become a suitable technique both for modeling and rendering complex 3D shapes. So, it seems interesting to convert other kind of models to a point or splat-based representations.

In this work, we present an approach to convert a parametric surface to an elliptical splat-based representation. Although this conversion supposes a loss of information going from an analytical to an approximate model, it will allow to locally modify zones with complex features, to mix surface and splat-based models and to take advantage of the existing point-based rendering methods.

The presented approach works in the parametric space and performs an adaptive sampling based on the surface curvature and a given error tolerance. The goal is to obtain an optimised set of elliptical splats that completely covers the surface. Two strategies are presented, one based on a quadrangular subdivision of the parametric space and the other on power Voronoi diagrams. Finally, some open problems are enumerated.

Obstacle Representation By Bump-Surfaces For Optimal Motion-Planning

Philip Azariadis
Univ. of the Aegean, Greece

This presentation describes a new method for global, near optimal, motion-planning of a robot (either mobile or redundant manipulator) moving in an environment cluttered with a priori known prohibited areas which have arbitrary shape, size and location. The proposed method is based on the novel notion of Bump-surfaces (or B-surfaces) which represent the entire robot environment through a single mathematical entity. The motion-planning solution is searched on a higher-dimension B-surface in such a way that its inverse image into the robot environment satisfies the given objectives and constraints.

The computed solution for a mobile robot consists of a smooth curve without self-loops which connects the starting and destination points with the shortest possible path. The same approach is also used for n-th degree-of-freedom manipulators where the end-effector reaches the destination position following a smooth short path avoiding the prohibited areas.

Several experiments are presented and discussed to illustrate the efficiency and effectiveness of the proposed motion-planning method in a variety of complex environments.

Accurate Detection of Ridges in Range Data

Alexander Belyaev
MPI für Informatik, Germany

A view-independent computational procedure for detecting surface creases in range data is proposed. The surface creases are defined via extrema of the principal curvatures along their corresponding curvature lines and, therefore, view-independent. A delicate analysis of curvature extrema properties leads to a simple modification of the well-known Canny non-maximum suppression. The modified non-maximum suppression scheme is rotationally-invariant and is used for simple and accurate detection of the surface creases.

Biorthogonal non-uniform spline wavelets and applications

Martin Bertram
TU Kaiserslautern, Germany

We present a lifting scheme for single-knot wavelets based on non-uniform B-splines as scaling functions. The method is used for the adaptive approximation of detailed functions, providing a sparse representation for compression and progressive transmission.

A Volume Approach to Model Repair and Smoothing

Pere Brunet
TU of Catalonia - Barcelona, Spain

The talk will describe a constrained fairing method for implicit surfaces defined on a voxelization. This method is suitable for computing a closed smooth surface that approximates an initial set of face connected voxels that approximate the surface of an object. The implicit surface is defined as the zero-set of a tensor-product uniform cubic Bspline. The fairing process is based on an objective function that tends to increase the B-spline continuity from C^2 to C^3 on the boundary faces of the voxels. The final surface is guaranteed to stab the predefined subset of voxels. The results have been used for error-bounded and topology-controlled model repair of complex geometric models in Cultural Heritage applications. Several examples will be presented and discussed.

Real time rendering of very large virtual reality scenes

Beat Brüderlin
TU Ilmenau, Germany

We present a visibility-guided approach for real-time visualization of very large scenes for interactive VR-applications. A run-time output-sensitive approach using among other things, spatial data structures and hardware occlusion tests has been developed to determine which parts of the scene will not have to be drawn at all, and where possibly lower resolution rendering suffices. An out-of-core rendering system with a two-level cache mechanism running on a PC is compared with a large shared memory architecture with multiple GPUs. We show that the performance (frame rate) depends very little on the number of polygons. We demonstrate the rendering system works with extremely large models consisting of several hundred million polygons.

Implicitization and Parameterization of Quadrics and Cubicoids by mu-bases

Falai Chen
Univ. of Science & Technology of China - Anhui

In this talk, we study the minimal mu-bases of quadric surfaces and cubic surfaces and we show that the minimal mu-bases are linear in the parametric variables. From the minimal mu-bases, we set up a complete transformation between parametric representations and implicit representations of quadrics and cubicoids.

Strong error bounds for piecewise linear interpolation of planar parametric curves

Wendelin Degen
Universität Stuttgart, Germany

Curves are commonly drawn by piecewise linear interpolation, but seldom one worries about the error. In the present paper we give a strong mathematical error analysis for curve segments with bounded curvature and length.Though the result seems very clear,the proof turned out to be unexpectedly hard, comparable to that of the famous four vertex theorem. Steiner's symmetrisation, methods of the calculus of variations and the compactness of equicontinuous sets of functions are the main tools to attain the goal.

Precise Voronoi Cell Extraction of Free-form Rational Planar Closed Curves

Gershon Elber
Technion - Haifa, Israel

We present an algorithm for generating the Voronoi cells for a set of rational C^1-continuous planar closed curves, which is precise up to machine precision. Initially, bisectors for pairs of curves, $(C(t), C_i(r))$, are generated symbolically and represented as implicit forms in the tr-parameter space. Then, the bisectors are properly trimmed after being split into monotone pieces. The trimming procedure uses the orientation of the original curves as well as their curvature fields, resulting in a set of trimmed-bisector segments represented as implicit curves in a parameter space.

A lower-envelope algorithm is then used in the parameter space of the curve whose Voronoi cell is sought. The lower envelope represents the exact boundary of the Voronoi cell.

Dimension of Spline Spaces over Unconstricted Triangulations

Gerald Farin
ASU - Tempe, USA

We consider a special class of 2D triangulations, namely those which have no subtriangulation where all boundary vertices have valence four or higher. For such triangulations, called unconstricted triangulations, one can define corresponding spline spaces consisting of piecewise polynomials of prescribed degree and smoothness. It is then possible – under certain simple conditions – to explicitly give the dimension of these spline spaces.

Algorithms for time-optimal control of CNC machines along curved tool paths

Rida T. Farouki
Univ. of California at Davis, USA

The problem of specifying the feedrate variation along a curved path, that yields minimum traversal time for a 3-axis CNC machine subject to constraints on the feasible acceleration along each axis, is addressed. In general, this time-optimal feedrate incurs "bang-bang control," i.e., maximum acceleration/deceleration is demanded of at least one axis throughout the motion. For a path defined by a polynomial parametric curve, we show that the (square of the) time-optimal feedrate can be determined as a piecewise-rational function of the curve parameter, with break-points corresponding to the roots of certain polynomial equations. Furthermore, this type of feedrate function is amenable to a real-time interpolator algorithm that drives the machine directly from the analytic curve description, eliminating the need for linear/circular G code approximations. The theoretical and computational aspects of such time-optimal feedrate functions are presented, together with experimental results from their implementation on a 3-axis mill driven by an open-architecture software controller.

The effect of parameterization on the approximation order of curve fitting

Michael S. Floater
University - Oslo, Norway

We obtain precise information about how the choice of parameterization affects the approximation order of curve interpolation based on polynomials, piecewise polynomials, and interpolatory subdivision curves. Chordal parameter values give full fourth order approximation for cubic interpolation, while uniform parameter values give only second order. More accurate approximations to arc length are required (and can be found) for higher degree interpolation.

Blossoming and the Lane-Riesenfeld Algorithm

Ron Goldman
Rice University, Houston, USA

A blossoming proof is presented for the Lane-Riesenfeld knot insertion algorithm for uniform B-splines.

Mesh-Based Inverse Kinematics

Craig Gotsman
Technion - Haifa, Israel

The ability to position a small subset of mesh vertices and produce a meaningful overall deformation of the entire mesh is a fundamental task in mesh editing and animation. However, the class of meaningful deformations varies from mesh to mesh and depends on mesh kinematics, which prescribes valid mesh cofigurations, and a selection mechanism for choosing among them. Drawing an analogy to the traditional use of skeleton-based inverse kinematics for posing skeletons, we define mesh-based inverse kinematics as the problem of finding meaningful mesh deformations that meet specified vertex constraints.

Our solution relies on example meshes to indicate the class of meaningful deformations. Each example is represented with a feature vector of deformation gradients that capture the affine transformations which individual triangles undergo relative to a reference pose. To pose a mesh, our algorithm efficiently searches among all meshes with specified vertex positions to find the one that is closest to some pose in a nonlinear span of the example feature vectors. Since the search is not restricted to the span of example shapes, this produces compelling deformations even when the constraints require poses that are different from those observed in the examples. Furthermore, because the span is formed by a nonlinear blend of the example feature vectors, the blending component of our system may also be used independently to pose meshes by specifying blending weights or to compute multi-way morph sequences.

C2 Filleting and Blending in Preliminary Aircraft Design

Thomas Grandine
The Boeing Company, Seattle, USA

Filleting and rounding operations have long been a staple of solid modeling. The classic filleting operation is the rolling ball fillet. This particular operation has the desirable property that it has constant cross sectional curvature, but it lacks both curvature continuity with the adjoining surfaces, and it doesn't always match the surfaces edges. This paper presents a modification of the rolling ball fillet which specifically addresses these two shortcomings.

Surfaces parametrized by their normals

Jens Gravesen
Technical University of Denmark

For a surface with non vanishing Gaussian curvature the Gauss map is regular and can be inverted. This makes it possible to use the normal as the parameter, and then it is trivial to calculate the normal and the Gauss map. This in turns makes it easy to calculate offsets, the principal curvatures, the principal directions, etc.

Such a parametrization is not only a theoretical possibility but can be used concretely. One way of obtaining this parametrization is to specify the support function as a function of the normal, i.e., as a function on the unit sphere. The support function is the distance from the origin to the tangent plane and the surface is simply considered as the envelope of its family of tangent planes.

Suppose we are given points and normals and we want a C^n-surface interpolating these data. The data gives the value of the support function at certain points (the given normals) on the unit sphere, and the surface can be defined by determining the support function as a C^{n+1} interpolant to the given values.

Fast Voronoi Modeling

Hans Hagen
DFKI Kaiserslautern, Germany

The common way to construct tessellations is to compare the distances between given reference points by a given distance function. To generalize this distance function concept we expand an existing approach which defines distance functions by their "unit circles". Our new approach allows modeling the "unit circles" by a closed NURBS, B-Spline, Bezier ... curve. Changing the control polygon directly affects the tessellation's appearance. To achieve interactive modeling we used a hardware accelerated rendering approach enabling us to use our NURBS distance representations as input of a growing process. This growing process easily takes into account weighting approaches like multiplicative, additive and even free functional weighting.

Hierarchical triangular splines

Stefanie Hahmann
LMC-IMAG - Grenoble, France

In this talk, we present a hierarchical triangular surface model. The surface is overall tangent plane continuous and is defined parametrically as a piecewise quintic polynomial. It can be adaptively refined while preserving the overall tangent plane continuity. This model enables designers to create a complex smooth surface composed of a small number of patches, to which details can be added by locally refining the patches until an arbitrary small size is reached. It is implemented as a hierarchical data structure where the top layer describes a coarse, smooth base surface, and the lower levels encode the details in local frame coordinates.

Volume Deformations in Action: A Forensic Reconstruction of George Washington

Dianne Hansford
ASU - Tempe, USA

The PRISM (Partnership for Research in Spatial Modeling) lab at Arizona State University is involved with a project to create three life-sized models of George Washington at ages 19, 45, and 57. Since most of the hard evidence of Washingtons appearance comes from approximately age 53, these models must be created using so-called forensic reconstruction methods. One aspect of this reconstruction is the deformation of a mandible for subsequent use in defining Washingtons facial appearance. We have employed volume deformations for this task. This talk will report on this novel use of a well-known tool, and report on the exciting and beneficial aspects of working with a multidiscipline team of researchers.

Generating a nice triangular mesh on a regular parametric surface

Victoria Hernandez-Mederos
ICIMAF - La Habana, Cuba

In this talk, we present an iterative algorithm to generate a nice triangular curvilinear mesh on a regular parametric surface. The main idea of our algorithm is to obtain an approximate reparametrization of the surface that behaves like a conformal map in a finite number of points. These points are the vertices of some nice planar triangulation T^{op} in the parameter space, that we would like to lift on the surface. Therefore, the image of T^{op} by means of the reparametrization provides us a triangular curvilinear mesh on the surface which reflects the nice properties of T^{op}.

A New Kind of Rational Interpolation

Kai Hormann
TU Clausthal, Germany

We present a new strategy for univariate global interpolation with a rational polynomial. Our interpolant does not have poles, reproduces cubic polynomials and numerical examples suggest that it has an approximation order of h^2. The method can be extended to match higher order derivative data which increases the degree of polynomial precision as well as the presumed approximation order.

Approximate parameterization of curves and surfaces

Bert Jüttler
University of Linz, Austria

We report on numerical techniques for parameterizing implicitly defined curves and surfaces by rational parametric representations. We have developed a predictor-corrector-type method, which is based on a combination of distance measures with functionals controlling the quality (inner geometry) of the parameterization.

Quantifying the Shape Effect of a Planar Control Point

Panagiotis Kaklis
National TU - Athens, Greece

We investigate the existence and analytic structure of the domain, where a planar control point is free to move while curvature retains sign. The obtained results are illustrated for planar Bézier curves.

G1 Surface Construction from Irregular Boundary Curve Network of a Ship Hull Form

Tae-Wan Kim
Seoul Nat. University, Corea

We present an algorithm for locally constructing a G1 surface of rectangular and triangular Bezier patches to interpolate given network of n-th degree Bezier curves. The curve network is assumed to consist of only 3- and 4 sided regions, and have 3, 4, and 5 valences at an interior node point. For many applications such as ship hull and car body design, it is important to interpolate the given curve network without any modification. However, most of previous researches need additional constraints on the curve

network at node points with even valences, which result in curve network modification. In this paper, we (a) adopt quartic scalar weight functions in the G1 continuity conditions for local interpolation with additional degree of freedom, and (b) derive new constraints on the weight functions at the node points with 4 valences for keeping the curve network unchanged. Moreover, we (c) propose a method for generating initial shape of piecewise surface patches using implicit functions. To demonstrate the quality of proposed algorithm, we included several examples of resulting smooth ship hullform surfaces generated from the curve network data of an actual ship.

Exact rational rolling-ball blends

Rimvydas Krasauskas
Vilnius University, Lithuania

The canal surface with the rational spine curve and rational radius function is known to be rational. We show that such parametrizations of canal surfaces are tightly related to the theory of ruled surfaces in RP^3. This insight allows to describe all such parametrizations with rational Gaussian image. The algorithm is developed for representing patches on a canal surface with given boundary curves by Bezier patches of minimal degree.

Applications include exact NURBS representations of several fixed or variable radius rolling ball blends between two natural quadrics, between cyclides/spheres or cyclides/planes.

On a Class of Weak Tchebycheff Systems

Tom Lyche
University - Oslo, Norway

In this talk we present a general study of spaces of the form $span < 1, x, \ldots, x^{n-2}, u(x), v(x) >$, where u and v are functions on an interval $[a, b]$ chosen so that the corresponding curve has good shape preserving properties. Examples of choices of u and v which have been considered in the literature are $e^{-\rho x}, e^{\rho x}$ on any interval, $\cos x, \sin x$ on an interval $[0, \alpha]$, and $(1-x)^\mu, x^\mu$ on the interval $[0, 1]$. The parameters ρ, α and μ, respectively are used as tension parameters to tighten a wiggly curve

Parametric Powell-Sabin Quasi-Interpolants

Carla Manni
Universit di Roma II, Italy

Quasi-interpolation is a general approach to construct, with low computational cost, efficient local approximants to a given set of data or a given function.

Quasi-interpolants are usually linear combinations of suitable positive, compactly supported basis functions (the “B-splines”) with coefficients which are linear functionals of various types (discrete, differential or integral). Recently, discrete quasi-interpolants (dQIs) of full approximation order in the space of C^1-quadratic splines on Powell-Sabin triangulations of a planar domain has been presented. In this talk, we describe the construction of parametric Powell-Sabin quadratic B-splines possessing tension properties and we present quasi-interpolants based on them. The constructed quasi-interpolants do not require derivatives and inherit the tension properties of the basis functions which allow an easy control of their shape.

Bivector Continuity

Stephen Mann
University of Waterloo, Canada

In geometric algebra, the outer product of two vectors yields a bivector, which represents the plane spanned by the two vectors. While a bivector has an area, it has no specfic shape, and may be thought of as reshapeable. In this talk, I investigate the idea of bivector continuity between triangular Bezier patches, and look at how it relates to parametric and geometric continuity.

Efficient video motion tracking using triangulations

Geraldine Morin
ENSEEIHT - IRIT Toulouse, France

This work presents a new triangular mesh tracking method preserving the mesh connectivity. Mesh motion estimation is an interesting tracking technique with applications in very low bitrate compression, object based coding and virtual view synthesis. Our method is a non-rigid generalization of the rigid template tracking learning-based method, proposed by Jurie and Dhome. Thanks to a learning step that can be done off-line, for a reasonable number of nodes, the tracking step can be performed in real-time.

Extended Subdivision Surfaces

Kerstin Müller
TU Braunschweig, Germany

An extended subdivision surface (ESub) is a generalization of Catmull-Clark and NURBS surfaces.

Depending on the knot intervals and valences of the vertices and faces, Catmull-Clark as well as NURBS patches can be generated using the extended subdivision rules. Moreover, an arbitrary choice of the knot intervals and topology is possible. Special features like sharp edges and corners are consistently supported by setting selected knot intervals to zero or by applying special rules. The refinement and limit-point rules for our non-uniform, non-stationary scheme are obtained via a new method using local Bézier control points. With our new surface it is possible to start with existing Catmull-Clark as well as trimmed NURBS models and to continue the modeling process using the extended subdivision options.

Quad-based Ternary Subdivision

Ahmad Nasri
American University of Beirut, Lebanon

Subdivision Schemes are now expanding from binary to ternary, introducing toplogical changes of the rules near the extraordinary vertices. The aim is to increase the smoothness of the limit surface such as achieving bounded curvature with convex hull property. The talk will present a ternary subdivision scheme for B-spline over arbitrary toplogy. In the non-uniform setting, it imporves on T-NURCCS, a Non-uniform Catmull- Clark subdivision with T-junctions, inspired by T-splines. In the uniform setting, it provides a global ternary subdivision for quad meshes.

Radial Hermite Operators

Gregory M. Nielson
ASU - Tempe, USA

A new technique for fitting scattered point cloud data with normal vectors is described. It is an implicit technique that is based upon the ideas of the modified quadratic Shepard (MQS) interpolant. This presentation concentrates on certain applications of these techniques including smoothing and approximate Boolean operations applied to general triangular mesh surfaces.

High-Quality Surfaces

Jörg Peters
University of Florida, USA

I will briefly talk about the (lack of) local linear independence of subdivision surfaces (joint work with Eric Wu) but speak mainly about two new ideas, Polar Parametrization and Guided Subdivision, developed with Kestas Karciauskas, in the context of high-quality fills of multi-sided holes. Polar Guided Subdivision yields curvature continuous surfaces of degree (4,3).

Integration of Computational Topology and Curve Subdivision

Thomas J. Peters
Univ. of Connecticut, USA

For spline curves embedded in 3-space of degree less than 5, it is shown that sufficiently many subdivisons will produce a control polygon that is ambient isotopic to the original curve. These can be space curves or planar curves and they may be open or closed. The use of isotopy for topological equivalence guarantees that these approximants will have the same embedding (and knot type) as the original curve. The result raises some interesting questions about many existing algorithms that employ subdivison for computer graphics and engineering simulations, as will be discussed.

Constrained 3D Shape Reconstruction Using a Combination of Surface Fitting and Registration

Helmut Pottmann
TU Wien, Austria

We investigate 3D shape reconstruction from measurement data in the presence of constraints. The constraints may fix the surface type or set geometric relations between parts of the object's surface, such as orthogonality, parallelity and others. It is proposed to use a combination of surface fitting and registration within the geometric optimization framework of squared distance minimization (SDM). In this way, we obtain a quasi-Newton like optimization algorithm, which in each iteration simultaneously registers the data set with rigid motion to the fitting surface and adapts the shape of the fitting surface. We present examples to show the applicability of our method to constrained 3D shape fitting for reverse engineering of CAD models and to high accuracy fitting with kinematic surfaces, which include surfaces of revolution (reconstructed from fragments of archeological pottery) and spiral surfaces, which are fitted to 3D measurement data of shells.

Morphing as Shadow Transitions

Hartmut Prautzsch
Universität Karlsruhe, Germany

Any two objects A and B can be viewed as two different projections of their cross product $A \times B$. Rotating and projecting $A \times B$ results in a continous transformation of A into B. During the rotation, the contour on the cross product remains the same although its projection changes. Based on this result, a fast and simple morphing algorithm has been implemented. Its characteristics are presented in the talk.

Global Roughness Indicator for Triangle Meshes

Anshuman Razdan
ASU - Tempe, USA

There are many methods and measures to describe shape in a local sense such as curvatures. Roughness has so far been defined in 1D sense on a surface (metrology applications) or in functional cases. We examine some ideas how to define this in a global sense and explore this measure as a function from local to global scale of a given triangle mesh.

On the smoothness of the fourpoint scheme

Ulrich Reif
TU Darmstadt, Germany

The fourpoint scheme with weights $[-w, 1/2+w, 1/2+w, -w]$ is the simplest non-trivial interpolatory subdivision scheme for curves. So far, only quite rough estimates on the range of parameters w yielding a C^1 scheme were known. In this talk, sharp bounds are derived thus solving a riddle of long standing.

Multisided Surface Patches in a Freely Designable Topology

Alyn P. Rockwood
Colorado School of Mines, USA

We describe a method to define multisided patches with any number of sides (2 to N) in a system that enables G2 continuity. The technology is based on finding the weighted least squares solution of points on given input curves where a separate parameter space with control structures determines its weights. It is a generalization of Shepard's method to a parameterized vector solution. The method generates surface patches that satisfy certain minimal energy conditions. It admits any parametric curve and points as controls for defining the surface.

Mean curvature subdivision

Malcolm A. Sabin
Numerical Geometry Ltd., Great Britan

The ambition is to make subdivision surfaces good enough to succeed NURBS as the next CAD/CAM standard for smooth surfaces. To do this we have to achieve two milestones. The first is to make them "good enough for Mercedes" (and it is a surprise that NURBS are considered good enough); the second is to achieve backwards compatibility with NURBS by supporting rationals and non-uniformity.

This talk addresses the first milestone by setting the target of reproduction of the natural quadrics. Geometric sensitivity can achieve reproduction of circles in a curve scheme: this was described at Tromso last year, in a scheme which modifies the four-point rules to position each new point to make the curvature vector there equal to the mean of those of its old neighbours. A non-stationary version of cubic spline subdivision can also be described in similar terms.

The analogue of this for surfaces seems to be to position the new points so that the mean curvature should be some average of the mean curvatures of surrounding points. If Floater's harmonic mean is used this can in principle give reproduction of cylinders, spheres and cones.

The talk will be a preliminary exploration of this territory, showing how only some of our familiar stationary subdivisions can be transformed into these terms, and exploring the issues which will almost certainly cause problems.

Smooth Discrete-Curves onto Clouds of Points: Algorithms and Applications in Digital Product-Design

Nikolaos S. Sapidis
Univ. of the Aegean, Greece

Point-based geometric models are currently investigated in many areas of "CAD & Computer Graphics" as they seem quite appropriate for swift computer- and graphic-processing. This paper focuses on Product Design based on point clouds, where a vital problem is drawing a "smooth discrete-curve" on an arbitrary point cloud. This problem is stated and analyzed, and an algorithmic solution is proposed based on projecting vertices on the given point-cloud. Theoretical and experimental results are presented

establishing the effectiveness of the proposed solution. The second part of the paper discusses application of the tool in industrial-design applications.

Modification of Surface and Solid Meshes: A Statistical Approach

Vladimir Savchenko
Hosei University - Tokyo, Japan

A method, based on an implementation of quasi-statistical modeling, for modification (improvement) of triangular, quadrilateral surface, and hexahedral solid meshes is discussed. The main intention is to attain a fairly smooth change from one mesh element to another without creating a significant difference between the shapes of neighboring elements. The geometry of the initial surface mesh is preserved by local mesh improving such that the new positions of the interior nodes of the mesh remain on the original discrete surface. Proposed method can be used in a pre-processing stage for subsequent studies (finite element analysis, computer graphics, etc.) by providing better input parameters for these processes and as an optimization step for mesh simplification. Experimental results are included to demonstrate the functionality of our approach.

Recent Results on Trivariate Macro-Elements

Larry Schumaker
Vanderbilt University - Nashville, USA

Given a tetrahedral partition $\triangle$ of a domain Ω, a C^r trivariate spline space is the space of piecewise polynomials functions defined on $\triangle$ that have r continuous derivatives on Ω. Such spaces are useful for dealing with a variety of problems in approximation, CAGD, data fitting, visualization, and the numerical solution of PDE's. The most useful of these spaces are defined via macro-elements, which provide a way of constructing a trivariate spline locally. But trivariate macro-element spaces are not easy to find, and except for the classical polynomial element, had been constructed only for the C^1 case. Here we report on recent work with Peter Alfeld in which we construct three new C^2 trivariate macro-element spaces.

Template-based Mesh Completion

Alla Sheffer
University of British Columbia - Vancouver, USA

Meshes generated by range scanners and other acquisition tools are often incomplete and typically contain multiple connected components with irregular boundaries and complex holes. This paper introduces a robust algorithm for completion of such meshes using a mapping between the incomplete mesh and a template model. The mapping is computed using a novel framework for bijective parameterization of meshes with gaps and holes. We employ this mapping to correctly glue together the components of the input mesh and to close the holes. The template is used to fill in the topological and geometric information missing in the input. The completed models are guaranteed to

have the same topology as the template. Further more, if no appropriate template exists or if only topologically correct completion is required a standard canonical shape can be used as a template. As part of our completion method we propose a boundary-mapping technique useful for mesh editing operations such as merging, blending, and detail transfer. We demonstrate that by using this technique we can automatically perform complex editing operations that previously required a large amount of user interaction.

Approximation of surfaces.
Restricted Delaunay Triangulation

Boris Thibert
UCBL - Lyon, France

In this talk I am interested in the approximation of the area (and the normals) of a smooth surface S with the area (and the normals) of a triangulation T which is close in the Hausdorff sense to the smooth surface. The quality of the approximation depends on the Hausdorff distance between T and S, an upper bound on the curvature of S and the shape of the triangulation T. This kind of result is very general and can be applied in surface reconstruction: In particular, I will show that the area of the restricted Delaunay triangulation T of an epsilon-sample on a smooth surface S is converging to the area of S, when epsilon tends to 0.

Normals of subdivision surfaces

Georg Umlauf
TU Kaiserslautern, Germany

While, for planar curves and bivariate functions, the cone of normals of a polynomial spline piece is enclosed by the cone of normals of its spline control polyhedron, the same is not true for surfaces, both at extraordinary points and in the regular, box-spline setting. A larger set, the cross products of control net edges, has to be considered.

Algebraic surface reconstruction from noisy point sets

Marek Vanco
TU Chemnitz, Germany

In the Digital Shape Reconstruction a digitized model of a scanned real 3D object is subdivided in coherent parts, which are represented with an appropriate surface. In our work we addressed the problem of creating an algebraic representation of an object digitized with a low-end scanner. We based the whole reconstruction process on a simple data structure - the neighborhood graph, which can be computed very efficiently. The nearest neighbors allow a fast estimation of local surface properties. The second order segmentation provides an initial subdivision of the object for a stable surface classification and fitting. In order to provide a reliable reconstruction of the point sets contaminated with high level noise, these have to be smoothed by a noise reduction procedure. Instead of creating an expensive and unreliable global triangulation, we modified and compared several mesh smoothing methods, so that they operate on k-nearest neighbors.

Segmentation and Surface Fitting in Digital Shape Reconstruction

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There is a rapidly growing demand to create digital replicas of existing 3D objects from measured data. Various approaches exist which differ in the amount of user interaction, computational efficiency and surface quality. The structure created over the polygonal mesh strongly influences the final CAD model. After discussing difficulties in segmentation two basic approaches are investigated in details (i) automatic (rapid) surfacing and (ii) functional decomposition. Several examples are given to show how existing algorithms can be improved.

Classification of Intersection Curves of Quadrics Surfaces in Real Projective Space

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We give complete classification of morphologies of the intersection curves of two quadrics (QSIC) in 3D real projective space. For each of QSIC morphologies we establish a characterizing algebraic condition expressed in terms of the number of real roots of the characteristic equation of two quadric surfaces, the multiplicities of these roots, and the signature of the pencil between and at these roots, with all this information encoded in an index sequence and the Segre characteristics. The key technique used for deriving these conditions is to analyze two simple quadrics obtained with Uhlig's simultaneous matrix diagonalization method.

An analytic pseudo-spectral method to generate a regular 4-sided PDE surface patch

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We describe a pseudo-spectral method for rapidly calculating an analytic approximation to a 4-sided PDE surface patch. The method generates an approximate solution consisting of three parts: an eigenfunction solution and a polynomial solution, both of which satisfy the generating partial differential equation exactly, and a third function, or 'remainder' term that ensures that the boundary conditions are satisfied exactly. Being analytic, the approximation allows an arbitary degree of surface refinement thereby facilitating physical analysis.

Computing curvature-based energy

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Curvature-based energy and forces are used in a broad variety of contexts, ranging from modeling of thin plates and shells to surface fairing and variational surface design.

The approaches to discretization preferred in different areas often have little in common: engineering shell analysis is dominated by finite elements, while spring-particle models are often preferred for animation and qualitative simulation due to their simplicity and low

computational cost. Both types of approaches have found applications in geometric modeling. While there is a well-established theory for

finite element methods, alternative discretizations are less well understood: many questions about mesh dependence, convergence and accuracy remain unanswered.

We discuss the general principles for defining curvature-based energy on discrete surfaces based on geometric invariance and convergence considerations. We show how these principles can be used to understand

the behavior of some commonly used discretizations, to establish relations between some well-known discrete geometry and finite element formulations and to derive new discretizations.