05391 Abstracts Collection
Algebraic and Numerical Algorithms and Computer-assisted Proofs
— Dagstuhl Seminar —

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Abstract. From 25.09.05 to 30.09.05, the Dagstuhl Seminar 05391 "Algebraic and Numerical Algorithms and Computer-assisted Proofs" was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. Links to extended abstracts or full papers are provided, if available.

Keywords. Self-validating methods, computer algebra, computer-assisted proofs, real number algorithms

05391 Executive Summary – Numerical and Algebraic Algorithms and Computer-assisted Proofs

The common goal of self-validating methods and computer algebra methods is to solve mathematical problems with complete rigor and with the aid of computers. The seminar focused on several aspects of such methods for computer-assisted proofs.

Keywords: Self-validating methods, computer algebra, computer-assisted proofs, real number algorithms

Joint work of: Buchberger, Bruno; Jansson, Christian; Oishi, Shin'ichi; Plum, Michael; Rump, Siegfried M.

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2006/454

Verification of Solutions for a Class of Nonlinear Complementarity Problems

Götz Alefeld (Universität Karlsruhe, D)

We present a computational enclosure method for the solution of a class of nonlinear complementarity problems. The procedure also delivers a proof for the uniqueness of the solution.

Keywords: Complementarity problems, verification of solutions

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2006/443

Solving and validating optimization problems by symbolic-numeric cylindrical algebraic computation

Hirokazu Anai (Fujitsu Laboratories - Kawasaki, J)

We present a successful example of symbolic-numeric approaches to computer-assisted proofs in constraint solving/optimization.

Our tool is cylindrical algebraic decomposition (CAD). CAD is a general-purpose symbolic method aiming for quantifier elimination (QE), which is a powerful tool to resolve optimization problems exactly (even for the case of non-convex and parametric optimization). First we propose a new faster algorithm to prove and solve semidefinite programming (SDP), which is a class of convex optimization problems with a linear objective function and linear matrix inequality (LMI) constraints, exactly by exploiting the convexity of the SDP feasibility domain based on CAD algorithm. This is achieved by combining a symbolic algorithm of CAD and a lifting strategy that takes into account the convexity properties of SDP. QE based on CAD is not practical on real computers, since CAD usually consists of many of purely symbolic computations and has bad computational complexity in nature. Therefore, we propose an efficient symbolic-numeric algorithm by using numerical computation particularly in handling algebraic numbers in CAD construction. This strategy greatly improves the efficiency, while this causes uncertainty of the computed results depending on accuracy of numerical computation. Hence we need an effective validation method of the results. We also provide machinery for validating the computed results by reconstructing them exactly with smaller number of symbolic computations. This is realized by using the symbolic information of algebraic numbers appeared in the whole computation. Moreover, we improve the efficiency of the symbolic computation in the reconstruction procedure by employing “dynamic evaluation".

Keywords: Optimization, symbolic-numeric, cylindrical algebraic decomposition

Joint work of: Anai, Hirokazu; Parrilo, Pablo; Yokoyama, Kazuhiro

Verified differential equation solution techniques for delayed differential equations and proving chaotic behavior

Balazs Banhelyi (University of Szeged, H)

We discuss two problems, related to differential equations. To provide a mathematical proof, we have to use validated solvers. In this case the verification means mathematical verification, i.e. rounding and other errors are considered.

In the first problem, we consider the following delayed differential equation:

$$y' = -\alpha \left(e^{y(t-1)} - 1 \right).$$

We are interested in checking whether for all $\alpha \in \left[\frac{3}{2}, \frac{\pi}{2}\right]$, there exists a unit length time segment where the absolute value of solution is less than 0.075.

Our technique applies multiple precision interval arithmetic and Taylor series. We proved the above statement for some tiny intervals around certain computer representable numbers, but we were not able to prove it for all points of the parameter interval.

In the second problem, we consider the forced damped pendulum ($y'' = -\sin(y) - 0.1y' + \cos(t)$).

This technique checks three geometrical conditions to be fulfilled by all points of the solution region. Our checking method applies interval arithmetic and a recursive subdivision technique.

We discuss the details of the proof, wherein we emphasize verification aspects. Finally we analyze the three conditions separately. With this method, we were able to prove an earlier conjecture on suspected chaotic region.

Joint work of: Bánhelyi, Balázs; Csendes, Tibor; Hatvani, László; Krisztin, Tibor; Garay, Barnabás

Analytical roots of an algebraic theorem

Prashant Batra (TU Hamburg-Harburg, D)

The fundamental theorem of algebra exhibits the total number of solutions of a complex non-linear algebraic equation. Many discussions of non-linear system-solving, complexity theory, topology and related fields start from this show-case problem.

Smale's approach to determine the total cost of algorithms of analysis considers root-finding via Newton's method. We will relate Smale's problems to works of earlier writers, and by tieing several loose ends give an algorithmic proof of the fundamental theorem via the Newton iteration. The proposed analytical scheme is of polynomial cost, and may be implemented using validated inclusions.

Keywords: Newton iteration, Fundamental theorem of algebra, purely iterative algorithm

Curve Veering for the Parameter-Dependent Clamped Plate

Henning Behnke (TU Clausthal, D)

The computation of vibrations of a thin rectangular clamped plate results in an eigenvalue problem with a partial differential equation of fourth order. If we change the geometry of the plate for fixed area, this results in a parameter-dependent eigenvalue problem. For certain parameters, the eigenvalue curves seem to cross. We give a numerically rigorous proof of curve veering, which is based on the Lehmann-Goerisch inclusion theorems and the Rayleigh-Ritz procedure.

Keywords: Parameter-dependent eigenvalue problem, guaranteed bounds, curve veering

Formal Proofs about DSPs

Sylvie Boldo (INRIA Futurs - Orsay, F)

DSPs are embedded processors that we use more than ever even without noticing them (washing machines, cars...). Their internal representation of floating-point numbers is often two's complement that is slightly different from the general-purpose processors's. We formalized a generic representation including both representations and proved many results both old and new about this representation. We then give some remarks on formal proofs and their possibilities.

Keywords: Formal proof, Two's complement

Lazy Thinking: A New Method for Algorithm Synthesis

Bruno Buchberger (Universität Linz, A)

We present a new method, called "lazy thinking", for the algorithmic synthesis of algorithms from formal specifications. The method proceeds by trying out various algorithm schemes for the given problem. An algorithm scheme is a recursive function definition whose subalgorithms are unspecified. For each scheme, an (automated) proof of the correctness of the scheme w.r.t. the given problem specification is attempted. Such an attempt will normally fail because nothing is known about the subalgorithms. However, from the failing proof attempt, by a "specification generation algorithm", we can often generate specifications for the subalgorithms that will make the correctness proof work. Now we can apply the lazy thinking method recursively to the subalgorithms until we arrive at specifications that are met by algorithms in the available algorithm library.

In the talk, we will demonstrate that, with this method, it is possible to synthesize non-trivial algorithms, e.g the author's Groebner bases algorithm. In fact, we will see that the crucial ingredient of this algorithm, the notion of S-polynomials, can be *invented* automatically by the lazy thinking method. The method has been implemented in the frame of the Theorema system.

Keywords: Groebner bases, algorithm synthesis, algorithm verification, correctness, automated reasoning

A new statistical test applied to a classical interval problem

Jean-Marie Chesneaux (Université Paris VI, F)

An important industrial problem is to determine the range of intermediate or final results comparatively to variations of data. For instance, when one wants to simulate the answer of an electronic system to an impulse, the uncertainties of the values of the components lead to relative errors of outputs. The knowledge of a realistic range for these errors is an important challenge for designers. In this case, only final results are concerned.

A second example is given by the use of numerical functionalities in embedded computations. Such functionality has to work for a range of data. Generally each data belongs to an interval. The designer wants to minimize the size of the representations for the variables and intermediate computations but without generating an overflow. In this case, all intermediate results are concerned.

It is a classical problem of interval arithmetic. We presents a new statistical test which allows to estimate ranges in both cases and its implementation in the SOFA toolbox.

Joint work of: Chesneaux, Jean-Marie; Dider Laurent-Stéphane; Rico, Fabien

A global optimization model for locating chaos

Tibor Csendes (University of Szeged, H)

We present a global optimization model to find chaotic regions of certain dynamic systems. The technique has two innovations: first an interval arithmetic based guaranteed reliability checking routine to decide whether an inclusion relation holds, and a penalty function based nonlinear optimization problem that enables us to automatize the search for fitting problem instances.

We provide the theoretical results proving correctness and convergence properties for the new algorithm. The talk discusses also the results achieved by the presented method.

Keywords: Chaos, global optimization, validated numerics

Joint work of: Csendes, Tibor; Bánhelyi, Balázs ; Garay, Barnabás

Full Paper:
http://www.inf.u-szeged.hu/~csendes/henon.pdf

Enclosure for the Biharmonic Equation

Borbála Fazekas (Universität Karlsruhe, D)

In this paper we give an enclosure for the solution of the biharmonic problem and also for its gradient and Laplacian in the L_2-norm, respectively.

Keywords: Biharmonic problem, enclosure, finite elements

Joint work of: Fazekas, Borbála; Plum, Michael; Wieners, Christian

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2006/448

Images of Julia sets that you can trust

Luiz Henrique de Figueiredo (IMPA - Rio de Janeiro, BR)

We present an algorithm for computing images of quadratic Julia sets that can be trusted in the sense that they contain numerical guarantees against sampling artifacts and rounding errors. Interval arithmetic is the main tool used to robustly classify rectangles in the complex plane as being on either side of the Julia set. The union of the regions that cannot be so classified is guaranteed to contain the Julia set.

Keywords: Julia sets, interval arithmetic

Joint work of: de Figueiredo, Luiz Henrique; Oliveira, João Batista

Squeezing the Most out of Interval Arithmetic for Spherical t-Designs for Quadrature on the Sphere

Andreas Frommer (Universität Wuppertal, D)

Spherical t-designs are quadrature rules for the sphere with equal weights and $(t+1)^2$ nodes, having a degree of exactness of t. It is not known whether such designs always exist and whether their distribution of nodes are attractive geometrically.

In this work we improve over previous attempts to computationally verify the existence of t-designs. Our approach is based on numerical verification of the existence of the quadrature nodes via Krawczyk's method and by using efficient as well as tight methods to bound function values via interval arithmetic. We arrive at unprecedented existence results for t-designs close to the extremal system for values of t as large as 50.

Keywords: Spherical t-designs, existence tests, interval arithmetic,result verification

Joint work of: Chen, Xiaojun; Frommer, Andreas; Lang, Bruno

Pseudozeros, stability radius and interval polynomials

Stef Graillat (Université de Perpignan, F)

The pseudozero set of a polynomial p is the set of all the zeros of all the polynomials that are near to p. Some explicit formulas have been provided to compute this set. In this talk, we give some applications of this notion. We present a numeric-symbolic algorithm to compute the stability radius of a polynomial. We give some numerical experiments.

An interval polynomial is a polynomial whose coefficients are real intervals.

The zeros of an interval polynomial are the zeros of polynomials with real coefficients belonging to the interval coefficients of the interval polynomial.

We show how to draw the zeros of interval polynomial thanks to the notion of pseudozeros. We present a Matlab tool to compute this set.

Keywords: Pseudozeros, stability radius, interval polynomials

Toward accurate polynomial evaluation in rounded arithmetic (short report)

Olga Holtz (Univ. California - Berkeley, USA)

Given a multivariate real (or complex) polynomial p and a domain $\mathcal{D}$, we would like to decide whether an algorithm exists to evaluate $p(x)$ accurately for all $x \in \mathcal{D}$ using rounded real (or complex) arithmetic.

Here "accurately" means with relative error less than 1, i.e., with some correct leading digits. The answer depends on the model of rounded arithmetic:

We assume that for any arithmetic operator $op(a, b)$, for example $a+b$ or $a \cdot b$, its computed value is $op(a, b) \cdot (1 + \delta)$, where $|\delta|$ is bounded by some constant ϵ where $0 < \epsilon \ll 1$, but δ is otherwise arbitrary. This model is the traditional one used to analyze the accuracy of floating point algorithms.

Our ultimate goal is to establish a decision procedure that, for any p and $\mathcal{D}$, either exhibits an accurate algorithm or proves that none exists. In contrast to the case where numbers are stored and manipulated as finite bit strings (e.g., as floating point numbers or rational numbers) we show that some polynomials p are impossible to evaluate accurately. The existence of an accurate algorithm will depend not just on p and $\mathcal{D}$, but on which arithmetic operators and constants are available to the algorithm and whether branching is permitted in the algorithm.

Toward this goal, we present necessary conditions on p for it to be accurately evaluable on open real or complex domains $\mathcal{D}$. We also give sufficient conditions, and describe progress toward a complete decision procedure. We do present a complete decision procedure for homogeneous polynomials p with integer coefficients, $\mathcal{D} = \mathbb{C}^n$, using only arithmetic operations $+$, $-$ and $\cdot$.

Keywords: Accurate polynomial evaluation, models or rounded arithmetic

Joint work of: Demmel, James; Dumitriu, Ioana; Holtz, Olga

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2006/447

Rigorous Results in Combinatorial Optimization

Christian Jansson (TU Hamburg-Harburg, D)

Many current deterministic solvers for NP-hard combinatorial optimization problems are based on nonlinear relaxation techniques that use floating point arithmetic.

Occasionally, due to solving these relaxations, rounding errors may produce erroneous results, although the deterministic algorithm should compute the exact solution in a finite number of steps. This may occur especially if the relaxations are ill-conditioned or ill-posed, and if Slater's constraint qualifications fail. We show how exact solutions can be obtained by rigorously bounding the optimal value of semidefinite relaxations, even in the ill-posed case. All rounding errors due to floating point arithmetic are taken into account.

Keywords: Combinatorial Optimization, Semidefinite Programming, Ill-posed Problems, Verification Methods

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2006/446

See also: C. Jansson, Termination and Verification for Ill-Posed Semidefinite Programming Problems, submitted

Construction of Validated Uniqueness Regions for Nonlinear Programs in which Convex Subspaces have been Identified

Baker Kearfott (Univ. of Louisiana - Lafayette, USA)

In deterministic global optimization algorithms for constrained problems, it can be advantageous to identify and utilize coordinates in which the problem is convex, as Epperly and Pistikopoulos have done. In self-validating versions of these algorithms, a useful technique is to construct regions about approximate optima, within which unique local optima are known to exist; these regions are to be as large as possible, for exclusion from the continuing search process. In this paper, we clarify the theory and develop algorithms for constructing such large regions, when we know the problem is convex in some of the variables. In addition, this paper clarifies how one can validate existence and uniqueness of local minima when using the Fritz John equations in the general case. We present numerical results that provide evidence of the efficacy of our techniques.

Keywords: Nonconvex optimization, global optimization, computational complexity, automatic differentiation, GlobSol

Full Paper:
http://interval.louisiana.edu/preprints/2005_convex_in_subspace.pdf

Lurupa - Rigorous Error Bounds in Linear Programming

Christian Keil (TU Hamburg-Harburg, D)

Linear Programming has numerous applications. Recently it has been shown that many real world problems exhibit numerical difficulties due to ill-conditioning.

Lurupa is a software package for computing rigorous bounds for the optimal value of a linear program. The package can handle point and interval problems. Numerical experience with the Netlib lp library is given.

Keywords: Linear programming, rigorous error bounds, Netlib lp library, interval arithmetic

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2006/445

On the global uniqueness of Stokes' wave of extreme form

Kenta Kobayashi (Kyushu University, J)

We present a proof of the global uniqueness of Stokes' wave of extreme form.

Stokes' wave is a water wave which forms a corner singularity at the crest, and considered to be a limit of positive solutions of Nekrasov's equation, which represents periodic gravity waves of permanent form on the free surface. The uniqueness of Stokes' wave of extreme form has been a longtime open problem, which we affirmatively prove with the aid of the numerical verification method.

Accurate and validated polynomial evaluation in floating point arithmetic

Philippe Langlois (Université de Perpignan, F)

Using error-free transformations, we improve the classic Horner Scheme (HS) to evaluate (univariate) polynomials in floating point arithmetic. We prove that this Compensated Horner Scheme (CHS) is as accurate as HS performed with twice the working precision. Theoretical analysis and experiments exhibit a reasonable running time overhead being also more interesting than double-double implementations. We introduce a dynamic and validated error bound of the CHS computed value. The talk presents these results together with a survey about error-free transformations and related hypothesis.

Keywords: Polynomial evaluation, Horner scheme, error-free transformation, floating point arithmetic, accuracy

Joint work of: Langlois, Philippe; Graillat, Stef; Louvet, Nicolas

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2006/442

Integration of reliable algorithms into modeling software

Wolfram Luther (Universität Duisburg-Essen, D)

In this note we discuss strategies that would enhance modern modeling and simulation software (MSS) with reliable routines using validated data types, controlled rounding, algorithmic differentiation and interval equation or initial value problem solver. Several target systems are highlighted. In stochastic traffic modeling, the computation of workload distributions plays a prominent role since they influence the quality of service parameters. INoWaTIV is a workload analysis tool that uses two different techniques: the polynomial factorization approach and the Wiener-Hopf factorization to determine the work-load distributions of GI/GI/1 and SMP/GI/1 service systems accurately. Two extensions of a multibody modeling and simulation software were developed to model kinematic and dynamic properties of multibody systems in a validated way. Furthermore, an interface was created that allows the computation of convex hulls and reliable lower bounds for the distances between subpaving-encoded objects constructed with SIVIA (Set Inverter Via Interval Analysis).

Keywords: Reliable algorithms, stochastic traffic modeling, multibody modeling tools, geometric modeling

Joint work of: Luther, Wolfram; Haßlinger, Gerhard; Auer, Ekaterina; Dyllong, Eva; Traczinski, Daniela & Holger

An Exact Descartes Algorithm for Polynomials with Bitstream Coefficients

Kurt Mehlhorn (MPI für Informatik, D)

We consider the root isolation problem for polynomials with real coefficients. We assume that coefficients are given as binary fractions with infinite places after the binary point. We describe a randomized variant of Descartes algorithm which isolates roots of square-free polynomials. For a n-th degree polynomial with leading coefficient greater than one, all coefficients bounded by 2^{τ}, and root separation ρ, the algorithm needs coefficient approximations to $O(n(log(1/\rho) + \tau))$ bits after the binary point and has an expected cost of $O(n^4(log(1/\rho) + \tau)^2)$ bit operations.

Full paper is available from author's web-page.

Eigenvalue enclosing and excluding method in gaps of essential spectrum

Kaori Nagatou (Kyushu University, J)

We consider an eigenvalue problem for Schroedinger operators with essential spectrum band. It is known that discrete spectrum can appear in a gap between two essential spectrum bands, but it seems that no one has succeeded to enclose or exclude an explicit eigenvalue in such a gap so far. In this talk we describe how to exclude an eigenvalue, how to find an approximate eigenpair and how to enclose an eigenpair in a gap with some numerical examples.

Keywords: Eigenvalue problem, essential spectrum

On numerical verification of solutions for three dimensional heat convection problems

Mitsuhiro T. Nakao (Kyushu University, J)

On the three dimensional heat convection, comparing with two dimension, more realistic and interesting bifurcation phenomena are observed in the actual problems in fluid mechanics. Our purpose is to verify numerically such a bifurcation structure in mathematically rigorous sense. Of course, the main difficulty comes from the fact that we could no longer use the formulation by using the stream function as in two dimensional case. Therefore, we have to consider a verification method which can be directly applied to the original Navier-Stokes equation. Under the stress free boundary condition for the velocity and Dirichlet boundary condition for the temperature, we define the appropriate function space satisfying the divergence free condition by using a Fourier spectral method. Then a solution can be formulated as a fixed point of some compact operator on that space. By considering the projection to the finite dimensional subspace as well as the constructive error estimates for the projection, we can formulate the verification procedure to get an enclosure of a bifurcating solution for concerned problems.

Global optimization and computer-assisted proofs

Arnold Neumaier (Universität Wien, A)

We present algorithmically useful necessary and sufficient conditions for global optimality of polynomial global optimization problems.

As an application, we present a branch and bound framework for rigorous reasoning with algebraic equations and inequalities over the reals, taking account of the effects of finite precision arithmetic. The implementation (in Matlab, on

top of the Intlab package) uses constraint propagation and the new optimality conditions; the latter are implemented using the semidefinite programming package SeDuMi. The problem input language is AMPL; the AMPL-to-Matlab interface is from the COCONUT environment.

Joint work of: Neumaier, Arnold; Schichl, Hermann; Domes, Ferenc

Fast and Accurate Computation of Sum and Dot Product

Takeshi Ogita (Waseda Univ. / JST - Tokyo, J)

Algorithms for summation and dot product of floating-point numbers are presented which are fast in terms of measured computing time. We show that the computed results are as accurate as if computed in twice or K-fold working precision, $K \geq 3$. For twice the working precision our algorithms for summation and dot product are some 40% faster than the corresponding XBLAS routines while sharing similar error estimates. Our algorithms are widely applicable because they require only addition, subtraction and multiplication of floating-point numbers in the same working precision as the given data. Higher precision is unnecessary, algorithms are straight loops without branch, and no access to mantissa or exponent is necessary.

Keywords: Accurate summation, accurate dot product, fast algorithms, verified error bounds, high precision

Joint work of: Ogita, Takeshi; Rump, Siegfried M.; Oishi, Shin'ichi

Full Paper:
http://epubs.siam.org/sam-bin/dbq/article/60181

See also: T. Ogita, S. M. Rump, S. Oishi: Accurate Sum and Dot Product, SIAM J. Sci. Comput., 26:6 (2005), 1955-1988.

Termination Guarantees for Algorithms that Solve Undecidable Decision Problems

Stefan Ratschan (MPI für Informatik, D)

A decision problem is a problem whose answer is either yes or no. If such a decision problem is undecidable, then no algorithm can terminate with an answer for all problem instances. In some cases, heuristic algorithms are available that work well in practice, but do not provide any formal guarantee of termination.

In the talk we will discuss two examples of undecidable problems, for which we have algorithms that are guaranteed to terminate with a definite answer for all problem instances, except for a well-specified class of degenerate cases. The algorithms are based on validated floating point arithmetic.

The first problem comes from the first-order theory of the real numbers, and the second problem occurs in the verification of hybrid dynamical systems.

Keywords: Real numbers, verification, hybrid systems

An Operator Algebra Approach to Linear Two-Point Boundary Value Problems

Markus Rosenkranz (RICAM - Linz, A)

We present a new symbolic approach to linear two-point boundary value problems that uses Groebner bases for determining the "Green's function" in an operator-algebra context. Up now, boundary value problems have not received much attention in symbolic computation since they may be subsumed under the heading of differential equations. However, standard methods from there will often not be effective due to the presence of the symbolic parameter. Moreover, it seems more natural to work directly on the level of operators since boundary value problems are really operator problems in disguise.

Keywords: Boundary Value Problems, Operator Algebra, Groebner Bases

How to Compute Largest Empty Anchored Cylinders in the Plane

Stefan Schirra (Universität Magdeburg, D)

In the largest empty anchored cylinder problem, the goal is to find a ray anchored at the origin that maximizes the minimum distance to a set of given points. Follert et al. presented an optimal $O(n \log n)$ algorithm for this problem. We show how to modify and implement their algorithms within the exact geometric computation paradigm. Surprisingly, for this problem, non-rational computations can be avoided at all.

Keywords: Exact geometric computation, computational geometry, algebraic degree of geometric problems

Guaranteed Tuning of Controllers

Eric Walter (Supélec - Gif-sur-Yvette, F)

Tuning a controller so that some specifications are guaranteed to be met is an important problem in automatic control theory, for which interval analysis turns out to be particularly useful. In this talk, tools recently developed for designing parametric controllers so as to achieve robust performance will be presented. The set of all controllers satisfying robust performance is characterized. Contrary to most of the existing methodology, this approach yields a guaranteed solution and avoids the problems of locality, such as the risk of getting trapped in local minima. The procedure is illustrated on the tuning of PID controllers, widely used in industry.

Keywords: Control, robust stability, PID, interval polynomials, Kharitonov theorem, set inversion

Joint work of: Walter, Eric; Bondia, Jorge; Kieffer, Michel

A computer assisted proof of a bifurcation point for the heat convection problems

Yoshitaka Watanabe (Kyushu University, J)

A computer assisted proof of the existence for a symmetry-breaking bifurcation point for the two-dimensional Rayleigh-Bénard convection is described. The method is based on the infinite dimensional fixed-point theorem using Newton-like operator. We will also propose a numerical verification algorithm which generates automatically on a computer a set including the exact bifurcation point. All discussed numerical examples are taken into account of the effects of rounding errors in the floating point computations.

Keywords: Computer assisted proof, bifurcation point, Rayleigh-Bénard convection

Formalization of mathematics

Freek Wiedijk (Radboud University of Nijmegen, NL)

I presented a short introduction to formalization of mathematics, using slides originally prepared for the TYPES summer school in Gothenburg this summer.

Keywords: Formalization of mathematics

Enclosures for variational inequalities

Christian Wieners (Univ. Karlsruhe, D)

We present a new method for proving the existence of a unique solution of variational inequalities within guaranteed close error bounds to a numerical approximation. The method is derived for a specific model problem featuring most of the difficulties of perfect plasticity. For this problem we summarize the existence properties and we present regularity results for the dual solution including estimates for the regularity at the boundary if the nonlinearity is restricted to the interior of the domain. Then we introduce a finite element method for the computation of admissible primal and dual solutions which guarantees the unique existence of a solution (by the verification of the safe load condition) and which allows for the determination of a guaranteed error bound. Finally, we present explicit existence results and error bounds in some significant configurations illustrating the derived a priori finite element error estimate as well as the obtained a posteriori bounds.

Joint work of: Wieners, Christian; Plum, Michael

A theorem for numerical verification of uniqueness

Nobito Yamamoto (The Univ. of Electro-Communications, J)

We introduce a theorem concerning numerical verification of uniqueness of the solutions to differential equations and integral equations. The equations are supposed to be transformed into fixed point equations with respect to Fre'chet differentiable operators. The theorem gives the existence and the local uniqueness of the solutions to the fixed point equations under certain conditions. Our talk will describe the theory and give an abstract of the proof. Numerical examples will be shown in order to illustrate how the theorem concerns validated computation for differential or integral equations.

Keywords: Numerical verification, uniqueness, fixed point equation, differential equation, integral equation

Joint work of: Yamamoto, Nobito; Nakao, Mitsuhiro T.; Watanabe, Yoshitaka

Attempts for efficient combination of numerical computation and symbolic computation

Kazuhiro Yokoyama (Rikkyo University - Tokyo, J)

We discuss how one can combine numerical computation and symbolic(algebraic) computation effectively and efficiently for breaking theoretical/computational difficulty. Here, as a promising example, we present a combined method for determining GCD of polynomials with parametric exponents. And we also give possible those combinations aiming for efficient computation of Galois groups and splitting fields of polynomials.

Keywords: Combination of numerical computation and symbolic computation