# Partial Behavioural Models for Requirements and Early Design

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**Abstract.** In this paper, we first motivate and summarize our recent work on creation, management, and specifically merging of partial behavioural models, expressed as model transition systems. We then address two issues coming out of MMOSS discussions: alphabet embedding as an alternative to common observational refinement and minimum refinement steps. We show that the former is not possible, and discuss informally how to define the latter.

## 1 Introduction and Goals of the Project

Event-based models such as Labeled Transition Systems (LTSs) [9] have been shown to be successful for modeling and reasoning about the behavior of software systems at the architectural level. These behavior models provide a basis for a wide range of successful automated analysis techniques, such as model-checking, animation, and simulation.

However, the adoption of behavior modeling and analysis technology by practitioners has been slow. Partly, this is due to the complexity of building behavioral models in the first place – behavior modeling remains a difficult, labor-intensive task that requires considerable expertise. In addition, and perhaps more importantly, the benefits of the analysis appear only at the end of a costly process of constructing a comprehensive behavior model. The reason for the latter is that traditional behavior models are required to be complete descriptions of the system behavior up to some level of abstraction, i.e., the transition system is assumed to completely describe the system behavior with respect to a fixed alphabet of actions. This completeness assumption is limiting in the context of software development process best practices which include iterative development, adoption of use-case and scenario-based techniques and viewpoint- or stakeholder-based analysis; practices which require modeling and analysis in the presence of partial information about system behavior.

Our aim is to address the limitations of existing behavior modeling approaches by shifting the focus from traditional behavior models to partial behavior models – models that are capable of distinguishing known behavior (both

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required and proscribed) from unknown behavior that is yet to be elicited. Our overall aim is to develop sound theory, techniques and tools that i) facilitate the construction of partial behavior models through model synthesis, ii) enable early feedback through partial behavior model analysis, and iii) support incremental elaboration of partial models.

The rest of this paper is organized as follows. We give an overview of our work in Section 2. Section 3 provides the formal background required to follow the discussions in the next two sections, which address two technical points that came out of the discussions during the Dagstuhl week: the need for observational refinement rather than standard refinement and alphabet embedding (Section 4), and the notion of a minimum refinement step (Section 5). Section 6 concludes the paper.

# 2 MTS Merge: A Foundation for Model Construction and Elaboration

In this section, we give an informal overview of our approach. Section 3 gives formal definitions for some of these concepts. Further details are available in [2, 13, 1].

Our approach adopts Modal Transition Systems (MTSs) [10] as the basis for describing partial system behavior. MTSs have been extensively studied and provide a formal underpinning for program analysis [8]. MTSs are a natural extension to Labelled Transition Systems (LTSs), which have been proven to be successful for modeling and analyzing the behavior of systems at the architecture level. Systems are modeled as a set of components or sub-systems that communicate and synchronize to provide system-level behavior. Each component is described as a transition system where labels on transitions represent an interaction of the component with the environment. In MTSs, each transition can be either 'required' or 'maybe'. The later means that it is not yet certain if the interaction modelled by the transition is required or prohibited in the final system. An MTS with no maybe-transitions is a model that is fully defined up to its alphabet, and hence corresponds to an LTS.

MTS models come equipped with a definition of refinement that captures the notion of "more defined than" or "more information than". A refinement step corresponds intuitively to removing maybe transitions or replacing them with a required one<sup>1</sup>. Refinement can be shown to preserve temporal properties [2,8]; hence, by refining an MTS, we are guaranteed that all properties that were true (false) in the partial model will continue to be so in the refinement. However, by iteratively refining an MTS into an LTS, all the temporal properties that are undecided in the partial model eventually become either true or false depending on the decisions made in the refinement process. An important practical result is that the value of a modal  $\mu$ -calculus property (true, false, or undecided) in an MTS can be obtained by performing two checks on LTSs derived from the

<sup>&</sup>lt;sup>1</sup> In fact, refinement is more subtle. We discuss it in Section 5.

MTS [2,8]. This means that two model-checks are sufficient to decide effectively whether a property holds or does not hold in all possible refinements of an MTS.

Although MTS models and refinement are an appropriate fit for incremental construction of models of software development artifacts, we identified that such models lack a specific concept that is necessary in the context of model elaboration, namely, *model merging*. Behaviour descriptions are typically provided by different stakeholders with different viewpoints [7], describing different, yet overlapping aspects [3] of the same system. How should these partial models be put together?

Composition of behaviour models is not a new idea [12,6]; however, its main focus has been on *parallel* composition which describes how two *different* components work together. In the context of model elaboration, what we are interested in is composing two partial descriptions of the *same* component to obtain a more elaborate version of both original partial descriptions. We call this operation a *merge*.

The core concept underlying MTS merging is that of a common observational refinement, which ensures that the required and proscribed behaviour of the models to be merged is preserved. Furthermore, merge should result in a model that is as least refined as possible, while still preserving the behaviour of the models being merged. The notion of common refinement also entails a natural definition of consistency as the existence of such common refinement.

We have studied MTS merge and produced a number of theoretical results and algorithms that support MTS construction and elaboration [2, 13]. Our results build on extensive existing research on MTSs (e.g., [8, 10, 11]). We have developed algorithms [13] for checking consistency of MTSs, merging consistent MTSs, and for approximating the merge of MTSs when the "best" merge is not unique and thus cannot be obtained algorithmically. We have characterized merge in terms of 3-valued modal  $\mu$ -calculus [2] and extended these results to temporal logics that are well suited for requirements specifications, such as the linear temporal logic of fluents (FLTL) [5]. In addition, because in practice MTS merging is likely to be combined with other operations over MTSs such as parallel composition, we have studied the algebraic properties of merging [2]. Finally, we have also started to study other refinement notions over MTS such as branching refinement [4].

### 3 Background

In this section, we give formal definitions of the key concepts that our work is based on: MTSs, alphabet, refinement, etc.

**Definition 1.** Let States be a universal set of states, Act be a universal set of observable action labels, and  $Act_{\tau} = Act \cup \{\tau\}$ . An LTS is a tuple  $P = (S, L, \Delta, s_0)$ , where  $S \subseteq$  States is a finite set of states,  $L \subseteq Act_{\tau}$  is a set of labels,  $\Delta \subseteq (S \times L \times S)$  is a transition relation, and  $s_0 \in S$  is the initial state. We use  $\alpha P = L \setminus \{\tau\}$  to denote the communicating alphabet (vocabulary) of P.

**Definition 2.** An MTS M is a structure  $(S, L, \Delta^r, \Delta^p, s_0)$ , where  $\Delta^r \subseteq \Delta^p$ ,  $(S, L, \Delta^r, s_0)$  is an LTS representing required transitions of the system and  $(S, L, \Delta^p, s_0)$  is an LTS representing its possible (but not necessarily required) transitions. We use  $\alpha M = L \setminus \{\tau\}$  to denote the communicating alphabet of M.

When depicting MTSs we label states for reference, we assume that they start in state 0. Maybe transitions are denoted with a question mark following the label, and transitions on sets are short for a single transition on every element of the set. For example, the MTSs  $\mathcal{I}$  with alphabet  $\{c\}$  and  $\mathcal{J}$  with alphabet  $\{a,b,c\}$  are shown in Figure 1.

Given an MTS  $M=(S,L,\Delta^r,\Delta^p,s_0)$  we say M transitions on  $\ell$  through a required transition to M', denoted  $M \xrightarrow{\ell} M'$ , if  $M'=(S,L,\Delta^r,\Delta^p,s_0')$  and  $(s_0,\ell,s_0') \in \Delta^r$ , and M transitions through a possible transition, denoted  $M \xrightarrow{\ell} M'$ , if  $(s_0,\ell,s_0') \in \Delta^p$ . Similarly, for  $\gamma \in \{r,p\}$  we write  $M \xrightarrow{\hat{\ell}} M'$  to denote that either  $M \xrightarrow{\ell} M'$  or  $\ell = \tau$  and P = P' are true, and we use  $P \xrightarrow{\ell} P'$  to denote  $P(\xrightarrow{\tau}_{\gamma})^* \xrightarrow{\ell} (\xrightarrow{\tau}_{\gamma})^* P'$ .

We capture the notion of elaboration of a partial description into a more comprehensive one using refinement and observational refinement:

**Definition 3.** (Strong Refinement) Let  $\wp$  be the universe of all MTSs. N is a refinement of M, written  $M \leq N$ , when  $\alpha M = \alpha N$  and (M, N) is contained in some refinement relation  $R \subseteq \wp \times \wp$ , for which the following holds for all  $\ell \in Act_{\tau}$  and all  $(M, N) \in R$ :

$$1. (M \xrightarrow{\ell}_{r} M') \Longrightarrow (\exists N' \cdot N \xrightarrow{\ell}_{r} N' \wedge (M', N') \in R)$$
$$2. (N \xrightarrow{\ell}_{p} N') \Longrightarrow (\exists M' \cdot M \xrightarrow{\ell}_{p} M' \wedge (M', N') \in R)$$

**Definition 4.** (Observational Refinement) N is an observational refinement of M, written  $M \preceq_o N$ , if  $\alpha M = \alpha N$  and (M, N) is contained in some refinement relation  $R \subseteq \wp \times \wp$  for which the following holds for all  $\ell \in Act_{\tau}$ :

1. 
$$(M \xrightarrow{\ell}_r M') \Longrightarrow (\exists N' \cdot N \Longrightarrow_r N' \land (M', N') \in R)$$
  
2.  $(N \xrightarrow{\ell}_p N') \Longrightarrow (\exists M' \cdot M \Longrightarrow_p M' \land (M', N') \in R)$ 

Two models are strongly (observationally) equivalent noted  $\equiv (\equiv_o)$  if they strongly (observationally) refine each other. We denote by M@X the result of restricting  $\alpha M$  to X, i.e., replacing actions in  $Act \backslash X$  with  $\tau$  and reducing  $\alpha M$  to X. We say that an LTS L is an implementation of an MTS M if N is a refinement of M.

We now review the process of merge. The intuition behind this process is to find a more precise system by combining what is known from two partial descriptions of that system. This is a process aimed at finding a common observational refinement, and may require human intervention [13].

**Definition 5.** An MTS P is a common refinement (CR) of MTSs M and N if  $\alpha P \supseteq (\alpha M \cup \alpha N)$ ,  $M \preceq_o P@\alpha M$  and  $N \preceq_o P@\alpha N$ .

We denote the set of CRs of models M and N by  $\mathcal{CR}(M, N)$ . Two MTSs, M and N, are *consistent* iff  $\mathcal{CR}(M, N) \neq \emptyset$ . In [13], it is argued that the merged model should not introduce unnecessary behaviours, and is therefore based on finding a *minimal common refinement*.

**Definition 6.** An MTS P is a minimal common refinement (MCR) of MTSs M and N if  $P \in \mathcal{CR}(M, N)$ ,  $\alpha P = \alpha M \cup \alpha N$ , and there is no MTS  $Q \not\equiv_o P$  such that  $Q \in \mathcal{CR}(M, N)$  and  $Q@\alpha P \preceq_o P$ .

For example, the models  $\mathcal{I}$  and  $\mathcal{J}$  in Figure 1 have two incomparable minimal common refinements,  $\mathcal{K}$  with alphabet  $\{c\}$  and  $\mathcal{L}$ , also shown in Figure 1. We denote the set of MCRs of models M and N by  $\mathcal{MCR}$ , so  $\mathcal{MCR}(\mathcal{I}, \mathcal{J}) = \{\mathcal{K}, \mathcal{L}\}$ .

## 4 Alphabet Embedding

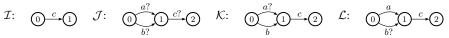
A major goal of our work is treatment of merge for models with different vocabularies. Such treatment allows integrating descriptions that have different scopes. Varying the scope of a description to include only the relevant aspects of the problem to be described makes model construction simpler. Consequently, it is common to find models describing different viewpoints provided by different stakeholders with different alphabets.

The way we have addressed merging models with different vocabularies is to define observational refinement and algorithms that construct models that are common observational refinements. A reasonable question to ask is why not unify vocabularies first, embedding each model into the unified vocabulary, and then merge the resulting systems which have the same vocabulary? Such an embedding may allow us to use the strong refinement and conjunction operator defined by Larsen [11] for MTSs with same alphabets. We first provide an informal argument as to why we believe such an embedding is not possible, and then show that even if it were possible, it does not resolve the major issues that merging models with different alphabets has.

Recall the models  $\mathcal{I}-\mathcal{L}$  shown in Figure 1 and described in Section 3. Starting with a model  $\mathcal{I}$  with an alphabet  $\{c\}$ , we would like to produce an embedding,  $\mathcal{I}'$ , with alphabet  $\{a,b,c\}$ , such that the minimal common refinements of  $\mathcal{I}'$  and  $\mathcal{I}$  are still  $\mathcal{K}$  and  $\mathcal{L}$ . The embedding should reflect the fact that  $\mathcal{I}$  does not say anything about labels a and b, hence the embedding  $\mathcal{I}'$  should not require nor proscribe the occurrence of a and b events. In other words, all states of  $\mathcal{I}'$  should always have maybe transitions on a and b.

A natural choice of embedding is to add transitions on the new labels from and to any state of  $\mathcal{I}$ , yielding the embedding  $\mathcal{I}'_1$  (see Figure 2). The problem with  $\mathcal{I}'_1$  is that it is inconsistent with  $\mathcal{I}$ . Note that the initial state of  $\mathcal{I}'_1$  has an outgoing required transition on c while the initial state of  $\mathcal{I}$  does not have possible transitions on c from it (i.e., it disallows c). Because of this, it is easy to see that there can be no common refinement of  $\mathcal{I}'_1$  and  $\mathcal{I}$ .

Thus, the embedding of  $\mathcal{I}$  should result in a model which in the initial state does not have a required c transition enabled. Yet, clearly, to preserve the semantics of  $\mathcal{I}$ , a state in which there is an enabled required c transition should



**Fig. 1.** Example MTS  $\mathcal{I}$  and  $\mathcal{J}$  and their minimal common refinements  $\mathcal{K}$  and  $\mathcal{L}$ . MC: Should J have c? or c



**Fig. 2.** Two candidate embeddings of MTS  $\mathcal{I}$ .

be reachable. Furthermore, this state should be reachable only through maybe transitions on a and b.

Consider a second candidate embedding  $\mathcal{I}'_2$  (see Figure 2) of  $\mathcal{I}$ . This model satisfies the criteria described above. However, note that  $\mathcal{J}$  is a common refinement of  $\mathcal{I}'_2$  and  $\mathcal{J}$ . Indeed, the combination of  $\mathcal{J}$  and  $\mathcal{I}'_2$  has lost the requirement that c must happen if a or b occur. Furthermore,  $\mathcal{J}$  is less refined that  $\mathcal{K}$  and  $\mathcal{L}$ . In conclusion, by embedding  $\mathcal{I}$  into the alphabet  $\{a,b,c\}$  as described by  $\mathcal{I}'_2$ , we have changed the semantics of merge.

We now address the general case. We shall refer to a state which has a required c transition enabled as a c state for brevity. We have already discussed why the initial state of the embedding of  $\mathcal{I}$  is not a c state. We also know that the embedding must have a reachable c state. If that state is reachable in strictly more than one a or b step, then the embedding has a state which can be reached by one a or b step which proscribes c. This would be inconsistent with  $\mathcal{I}$ . Suppose then that all c states are reachable in exactly one a or b step. If any of these steps are required, then the embedding will rule out either  $\mathcal{K}$  or  $\mathcal{L}$ . If neither are required, we have  $\mathcal{I}'_2$  which is not appropriate either. Hence, we can conclude that it is not possible to embed  $\mathcal{I}$  into  $\{a,b,c\}$  while preserving the minimal common refinements of  $\mathcal{I}$  and  $\mathcal{J}$ .

This argument above leads to the following theorem.

**Theorem 1.** There does not exist an embedding function  $f: MTS \times 2^{Act} \Rightarrow MTS$  that satisfies the condition that for all MTSs A and B,  $\mathcal{MCR}(A, B) = \mathcal{MCR}(f(A, \alpha A \cup \alpha B), f(B, \alpha A \cup \alpha B))$ .

#### 5 Refinement

Another issue that was raised at the workshop was about refinement: If the intuition of refinement is simply removing a maybe transition or replacing it with a required one, how can refinement capture the elaboration process that starts with an MTS that knows nothing about its alphabet to an LTS of the system to be. For instance, how can an MTS such as that in Figure 3(a) ever be refined to describe a set of traces that requires more than one MTS state? The answer is simple. The MTS in Figure 3(a) is equivalent to the one in Figure 3(b) (which has two states) and similarly equivalent to an MTS with an arbitrary number of states!

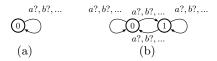
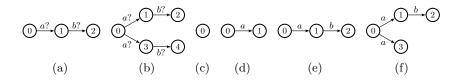


Fig. 3. Exploding states in MTS models



**Fig. 4.** Two equivalence MTS models (a and b), with all their implementations (c, d, e and f)

Having looping transitions as in Figure 3(a) is not necessary for replicating states. Consider Figure 4(a), this model is equivalent to Figure 4(b). In general it is simple to generate equivalent MTS by simply replicating branches of the MTS and introducing non-determinism.

Figure 4(a) is an interesting model to use to address some erroneous intuitions that one might have regarding MTS refinement. Figure 4(a), seems to provide two decision points, one for a and one for b, which may lead us to think that it has only three implementations Figure 4(c), (d), and (e). Yet because Figure 4(a) is equivalent to Figure 4(b), it also clearly admits the implementation Figure 4(f). Indeed, the intuition of making a maybe transitions required or removing it as the only refinement options that are available is deceiving.

Since refinement is not simply removal or replacement of maybe transitions with required ones, a reasonable question to ask is what are the strict refinement steps that can be used to generate all possible refinements of an MTS. Or if it is possible to transform an MTS such that all its refinements can be produced by simple removal or replacement of maybe transitions with required ones.

The latter seems possible for some notions of refinement (note that the preceding discussion in this sections applies to strong refinement). For example, consider the model in Figure 5(a). Its maybe transition can be refined to false and to true, resulting in the models in Figure 5(b) and (c) respectively. However, the models in Figure 5(d) and (e) are also observational refinements of Figure 5(a). These additional models either sometimes provide a, or sometimes inevitably provide a. Note that all four models (Figure 5(b), (c), (d) and (e)) are incomparable using MTS observational refinement.

Consider now the model in Figure 5(f). This model is observationally equivalent to Figure 5(a) and can be refined into any of its refinements by making binary choices on  $\tau$  transitions.

Is it possible then to transform an arbitrary MTS M into an equivalent one in which all decisions needed to reach an arbitrary refinement N are binary over the

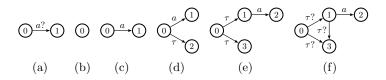


Fig. 5.

maybe transitions of M? The answer to this question is negative. Consider the refinements of Figure 3(a). What size of state space should the transformation of Figure 3(a) have considering that the refinements of Figure 3(a) have an unbounded number of states?.

Our final example refers to the question of whether it is possible to consider a notion of minimal refinement step or one-step refinement that allows generating all possible refinements of an MTS. We define one-step refinement as follows:

**Definition 7.** We say that MTS N is a one step refinement of M  $(M \leq_s N)$  if  $\forall P \cdot M \leq P \leq N, P \equiv M \vee P \equiv N$ .

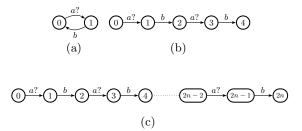
We can now reformulate the question of above as the following proposition

**Proposition 1.** For all MTSs M and N:  $M \leq N \wedge M \not\equiv N \implies \exists P_1 \cdots P_n \cdot M \equiv_o P_1 \leq_s P_2 \leq_s \cdots \leq_s P_n \equiv_o N$ .

We conjecture that the proposition above is false. Consider the MTS of Figure 6(a), this model admits refinements that require a b if an a transition has been taken. The MTS of Figure 6(b) is a refinement of Figure 6(a). The question then are the minimal refinement steps that lead from Figure 6(a) to Figure 6(b). The answer to this question, we conjecture, is negative. All the MTS models of the form Figure 6(c) are strict refinements of Figure 6(a) and it is also the case that Figure 6(b) is a strict refinement of all the MTS models of the form Figure 6(c) with n > 2. We conjecture that it is not possible to find a strict refinement of Figure 6(a) such that it is also a strict refinement of all MTS of the form of Figure 6(c). As a consequence, it is not possible to define a finite set of strict refinement steps that will allow us to generate each refinement of Figure 6(a).

#### 6 Conclusion

In this paper, we gave a high level overview of the process of merging partial behavioural models. We have also identified and discussed two problems that arose in this context during the MMOSS week: whether alphabet embedding provides an easy and natural way for merging MTSs with different vocabularies, and what is the intuition behind refinement and "minimum refinement step".



**Fig. 6.** There are no small step refinements that refine (a) into (b). There exist an infinite number of distinct refinements between (a) and (b) of the form of (c).

In addition to the aspects described here, we are working on the problem of synthesizing partial behavioural models from a collection of positive and negative scenarios and temporal properties; giving user support for choosing the best common refinement out of a set of possible MCRs; helping deal with inconsistency resolution, and many others. Overall, our goal is to provide a set of techniques, tools and methodologies to support all aspects of engineering with partial behavioural models.

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