06391 Abstracts Collection
Algorithms and Complexity for Continuous Problems
— Dagstuhl Seminar —

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Abstract. From 24.09.06 to 29.09.06, the Dagstuhl Seminar 06391 "Algorithms and Complexity for Continuous Problems" was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Computational complexity, partial information, high-dimensional problems, operator equations, non-linear approximation, quantum computation, stochastic computation, ill posed-problems

06391 Summary – Algorithms and Complexity of Continuous Problems

The seminar was devoted to the branch of computational complexity that studies continuous problems for which only partial information is available. As an important example we mention an operator equation $Lx = y$: here the right-hand side y and the coefficients of the (differential or integral) operator L are functions on some domain. These functions may only be evaluated at a finite number of properly chosen knots for the approximate computation of the solution x. Any such information about the coefficients is partial in the sense that it typically does not determine the solution x exactly.

The 8th Dagstuhl Seminar on Algorithms and Complexity of Continuous Problems attracted 50 participants from Computer Science and Mathematics, representing 11 countries and 4 continents. Among them have been 19 young researchers, some of whom have just received there diploma or master degree.

There were 43 presentations covering in particular the following topics:

- complexity and tractability of high-dimensional problems,
- complexity of operator equations and non-linear approximation,
- quantum computation,
- complexity of stochastic computation and quantization, and
- complexity and regularization of ill-posed problems,

together with applications in financial engineering and computer graphics. Abstracts are included in these Seminar Proceedings.

In addition to the substantial number of young participants another key feature of the seminar was the interaction between scientists working in different areas, namely, numerical analysis and scientific computing, probability theory and statistics, number theory, and theoretical computer science. In particular, distinguished researchers from numerical analysis were invited, and the mutual exchange of ideas was very inspiring and created many new ideas. Especially, one of the most challenging features of modern numerical analysis is the treatment of high-dimensional problems which requires several new paradigma. It has turned out that many developments that have been achieved in the IBC-community such as high-dimensional quadrature etc. will probably play a central role in this context, so that merging together the different approaches and ideas will be a very exciting topic in the near future.

Moreover, the meeting helped us to create new and to maintain the already existing various collaborations. Some ideas devoloped at the meeting have already flown into joint applications for reseach grants.

In a special event we have celebrated Henryk Wo'zniakowski, who has had his 60th birthday in 2006. Furthermore, Friedrich Pillichshammer has received the Information-based Complexity Young Researcher Award 2005, and Leszek Plaskota was the recipient of the 2006 Prize for Achievements in Information-based Complexity.

The participants of the seminar have been invited to submit a full paper to a Festschrift Issue of the Journal of Complexity

Financial support for a number of participants was granted by the German Research Foundation (DFG).

The organizers would like to thank all the attendees for their participation, and the Dagstuhl team for the excellent working environment and the hospitality at the Schloss.

Semidefinite programming characterization and spectral adversary method for quantum complexity with noncommuting unitary queries

Howard Barnum (Los Alamos National Laboratory, USA)

Generalizing earlier work characterizing the quantum query complexity of computing a function of an unknown classical "black box" function drawn from some set of such black box functions, we investigate a more general quantum query model in which the goal is to compute functions of $N \times N$ "black box" unitary matrices drawn from a set of such matrices, a problem with applications to determining properties of quantum physical systems.

We characterize the existence of an algorithm for such a query problem, with given query and error, as equivalent to the feasibility of a certain set of semidefinite programming constraints, or equivalently the infeasibility of a dual of these constraints, which we construct. Relaxing the primal constraints to correspond to mere pairwise near-orthogonality of the final states of a quantum computer, conditional on the various black-box inputs, rather than bounded-error distinguishability, we obtain a relaxed primal program the feasibility of whose dual still implies the nonexistence of a quantum algorithm. We use this to obtain a generalization, to our not-necessarily-commutative setting, of the "spectral adversary method" for quantum query lower bounds.

Keywords: Quantum query complexity, semidefinite programming

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/876

Complexity of the Schrödinger equation with finite-order weights

Arvid Bessen (Columbia University, USA)

We study the tractability of the evolution problem for the Schrödinger equation with an arbitrary, but fixed, potential function in the worst case. This problem is related to the question of whether a quantum computer can be simulated on a classical computer. We show that the problem is intractable if we allow arbitrary starting states as input. Therefore we restrict ourselves to starting states with bounded kinetic energy modelled as a weighted reproducing kernel Hilbert space, where the weights are product or finite-order weights. For product weights that are decaying sufficiently fast we can establish conditions for tractability and strong tractability, but are not able to allow symmetric functions as starting states. For finite-order weights we have tractability for all possible weights and are able to treat symmetric starting states.

Keywords: Schrödinger equation, tractability, weighted RKHS

Lower bound for average-case complexity of optimization for Brownian bridge with adaptive stopping rules

Jim Calvin (New Jersey Institute of Technology, Newark, USA)

We consider the problem of approximating the global minimum of a continuous function using sequentially chosen function evaluations. The error is analyzed in the average case for the Brownian bridge. If a fixed number n of function evaluations is allowed, then for any algorithm the error is bounded below by $\alpha \exp(-\beta n / \log(n))$ for some positive constants α, β. If adaptive stopping rules are allowed, let T_ϵ denote the random time until the conditional expected error is at most ϵ. Then there is a positive constant γ such that $ET_\epsilon \geq \gamma \cdot \log\log(1/\epsilon) \cdot \log(1/\epsilon)$.

Keywords: Global optimization, average complexity

Balancing principle for solving naturally linearized elliptic Cauchy problem

Hui Cao (Radon Institute for Computational and Applied Mathematics, Linz, A)

A classical ill-posed problem elliptic Cauchy problem is considered. By a natural linearization we transform the elliptic Cauchy problem into a linear ill-posed operator equation. A discretization is applied as a regularization method (also known as self-regularization) to obtain a stable approximate solution. The alancing principle as an adaptive strategy is studied to choose appropriate discretization level. Numerical tests illustrate the theoretical results.

Keywords: Cauchy problem, self-regularization, balancing principle

Quasi-Monte Carlo integration of functions of unbounded variation

Ronald Cools (Katholieke Universiteit Leuven, B)

It is well known that quasi-Monte Carlo methods for integration can work for functions of unbounded variation. In this talk we analyse the behaviour of nets and lattice rules for some discontinuous functions in 2 variables. For specific sequences of rules we can prove convergence order resp. $N^{-1/2}$, $N^{-3/4}$ and N^{-1}.

Keywords: Quasi-Monte Carlo

Joint work of: Ronald Cools, Tim Pillards

Nonlinear approximation of stochastic processes

Jakob Creutzig (Technische Universität Darmstadt, D)

We study free–knot spline approximation of stochastic processes using a (fixed or random) number of free knots. Our interest is in the asymptotics of the minimal error obtainable using n knots (on the average) as $n \to \infty$. For examples including the (fractional) Brownian Motion, the symmetric stable Levy process, and autonomous scalar diffusions, rates of convegence, and, in some cases, strong asymptotics of this error are established.

Keywords: Nonlinear approximation, (fractional) Brownian motion, Lévy processes, diffusion processes

Joint work of: Jakob Creutzig, Mikhail Lifshits, Thomas Müller–Gronbach, Klaus Ritter

A taste of compressed sensing

Ronald A. DeVore (University of South Carolina, USA)

Compressed Sensing has its roots in results on Gelfand widths from the late 1970's. It is intimately connected with the fundamental questions in Information Based Complexity. It deals with the problem of encoding signals which are known to be sparse or compressible with respect to a given basis. The problem is to take as few samples as possible of the signal while obtaining enough information to recover the signal to a prescribed accuracy. Here a sample is to be interpreted as the application of a linear functional to the signal.

Compressed Sensing has received a revival with the recent results of Candes-Tao and Donoho which give practical algorithms for sampling (encoding) and decoding. We shall discuss two topics. The first is the best estimates we can give for recovering a signal given a budget of n samples. The second is the role of randomness in sampling and the role of probabilty in estimating performance.

Keywords: Compressed sensing, Gelfand widths, information based complexity, instance-optimal

Quasi-Monte Carlo rules achieving arbitrary high convergence order

Josef Dick (University of New South Wales, AU)

In this talk we present first explicit constructions of point sets in the s dimensional unit cube yielding quasi-Monte Carlo algorithms which achieve the optimal rate of convergence of the worst-case error for numerically integrating high dimensional functions with square integrable partial mixed derivatives up to order $\delta \geq 1$ in each variable. The convergence is of $\mathcal{O}(N^{-\min(\delta,d)}(\log N)^{s\delta+1})$ for every $\delta \geq 1$, where d is a parameter of the construction which can be chosen arbitrarily large and N is the number of quadrature points. This convergence rate is known to be best possible up to some $\log N$ factors. We prove the result for the deterministic and also a randomized setting. The construction is based on a suitable extension of digital (t, m, s)-nets over finite fields of prime-power order.

Keywords: Numerical integration, quasi-Monte Carlo, digital nets, digital sequences

On discrete-time approximation of BSDEs with non-Lipschitz terminal condition

Stefan Geiss (University of Jyväskylä, FIN)

Backwards stochastic differential equations with non-Lipschitz terminal conditions are of interest for example in Stochastic Finance, where the terminal condition can be interpreted as pay-off function and one wishes to consider options like the Binary option, which is a pro-type of a non-Lipschitz pay-off.

The approximation error of discrete-time approximations of BSDE's has typically two sources: the error which occurs from the discretization of the forward component and the error which originates from the discretization of the backwards component. In this talk we discuss the backwards part and show that under fractional smoothness assumptions on g in terms of Malliavin Besov spaces one gets for the discretization of the backwards part the same asymptotic upper bound $1/\sqrt{n}$ for the L_2-error as in the case that g is Lipschitz provided that the equidistant time nets are replaced by special non-equidistant time nets chosen according to the degree of fractional smoothness of g.

Keywords: Backward stochastic differential equation, non-equidistant time discretization

Joint work of: Christel Geiss, Stefan Geiss

Generalized tractability of linear tensor product problems -the restricted setting

Michael Gnewuch (Universität Kiel, D)

Many papers study polynomial tractability for multivariate problems. Let $n(\epsilon, d)$ be the minimal number of information evaluations needed to reduce the initial error by a factor of ϵ for a multivariate problem defined on a space of d-variate functions. Here, the initial error is the minimal error that can be achieved without sampling the function. Polynomial tractability means that $n(\epsilon, d)$ is bounded by a polynomial in ϵ^{-1} and d and this holds for all $(\epsilon^{-1}, d) \in [1, \infty) \times IN$.

In this talk we discuss generalized tractability by verifying when $n(\epsilon, d)$ can be bounded by a power of $T(\epsilon^{-1}, d)$ for all $(\epsilon^{-1}, d) \in \Omega$, where Ω can be a proper subset of $[1, \infty) \times IN$. Here T is a tractability function, which is non-decreasing in both variables and grows slower than exponentially to infinity. In particular we consider the set $\Omega = [1, \infty) \times \{1, 2, \ldots, d^*\} \cup [1, \epsilon_0^{-1}) \times IN$ for some $d^* \geq 1$ and $\epsilon_0 \in (0, 1)$. The focus of the talk is on linear tensor product problems for which we can compute arbitrary linear functionals as information evaluations. We present necessary and sufficient conditions on T such that generalized tractability holds for linear tensor product problems. We show some examples for which polynomial tractability does not hold but generalized tractability does.

Keywords: Multivariate problems, tensor product problems, tractability

Joint work of: Michael Gnewuch, Henryk Woźniakowski

On the complexity of searching maximum of a function on a quantum computer

Maciej Goćwin (AGH University of Science & Technology, Krakow, PL)

We deal with a problem of finding a maximum of a function from the Hölder class on a quantum computer. We show matching lower and upper bounds on the complexity of this problem. We prove upper bounds by constructing an algorithm that uses the algorithm for finding the maximum of a discrete sequences. To prove lower bounds we use results for finding logical OR of sequence of bits. We show that quantum computation yields a quadratic speed-up over deterministic and randomized algorithms.

Keywords: Numerical optimization, optimal algorithm, quantum computing, query complexity

Randomized information complexity of elliptic PDE: The L_p-case

Stefan Heinrich (Technische Universität Kaiserslautern, D)

We continue the study of randomized information complexity of elliptic partial differential equations with smooth coefficients in smooth domains D of R^d. The right hand side is supposed to belong to the Sobolev space $W_p^r(D)$, the solution is sought on a smooth d_1 dimensional submanifold M of D, and the error is measured in the norm of $L_p(M)$, where $r \in N$ and $1 \leq p < \infty$.

We obtain matching up to logarithmic factors upper and lower bounds. The results extend previous investigations of the $p = \infty$ case, which essentially used this assumption in the design of the algorithm. The new algorithm works for $1 \leq p < \infty$ and takes care of the effects related to the respective trace operator and to possible non-embedding of $W_p^r(D)$ into $C(D)$.

Keywords: Elliptic PDE, randomized information complexity

Cubature formulas with few points for symmetric measures

Aicke Hinrichs (Universität Jena, D)

We study cubature formulas for d-dimensional integrals with an arbitrary symmetric weight function of product form. We present a construction that yields a high polynomial exactness: for fixed degree $\ell = 5$ or $\ell = 7$ and large dimension d the number of knots is only slightly larger than the lower bound of Möller and much smaller compared to the known constructions. We also show, for any odd degree $\ell = 2k + 1$, that the minimal number of points is almost independent of the weight function. This is also true for the integration over the (Euclidean) sphere.

Keywords: High-dimensional integration, polynomial exactness

Joint work of: Aicke Hinrichs, Erich Novak

Almost tight bounds on the randomized and quantum complexity of initial-value problems

Bolesław Z. Kacewicz (AGH University of Science & Technology, Krakow, PL)

We report a progress in studying the complexity of initial-value problems (IVPs) in the randomized and quantum settings. The best known until 2005 upper and lower complexity bounds were presented by the author at the FoCM'05 conference. Since these bounds did not match, the complexity in the two settings was not known.

In this talk we present recent results, showing that the gap between the bounds can essentially be removed. Almost matching upper bounds are obtained by designing new randomized and quantum algorithms for IVPs. We discuss basic technical details of the algorithms design, and present crucial points of the performance analysis. It turns out that the solution of IVPs in the randomized and quantum settings is essentially as difficult as scalar integration.

Keywords: Initial-value problems, randomized and quantum settings, complexity

Towards simulating Markov chains by using rank-1 lattice sequences

Alexander Keller (Universität Ulm, D)

Instead of reducing Fredholm integral equations of the second kind to sums of integrals by using the Neumann series, it is also possible to interpret the transport equation as an iteration. Opposite to sums of integrals the dimension remains the dimension of the integral equation. Following this idea, Lecot developed a series of quasi-Monte Carlo methods and observed noticeable improvement. We analyze the deterministic scheme presenting simplifications and intuition why the scheme is working so well. We generalize the algorithms to using classical low discrepancy sequences and extend it for splitting and Russian roulette absorption. The simplest version of the algorithm can take advantage of rank-1 lattice sequences. We finally point out the interesting point of maximizing minimum distance, when selecting point sequences.

Keywords: Quasi-Monte Carlo, integrals equations

Digital nets and multivariate approximation

Peter Kritzer (Universität Salzburg, A)

We study the approximation of functions from high dimensional Walsh spaces using digital nets or polynomial lattice rules. The Walsh spaces are characterized by the rate of decay of the Walsh coefficients of the functions. These functions are approximated by Walsh polynomials whose Walsh coefficients are found by numerical integration using certain digital nets. We consider several ways of choosing the point sets used in our algorithm, including digital nets, polynomial lattice points constructed for integration, and polynomial lattice points specially constructed for approximation.

Keywords: Walsh space, function approximation, numerical integration, digital nets

Joint work of: Josef Dick, Peter Kritzer, Frances Kuo

Lattice-Nystrom method for Fredholm integral equation of the second kind

Frances Kuo (University of New South Wales, AU)

We consider Fredholm integral equations of the second kind of the form $f(\boldsymbol{x}) = g(\boldsymbol{x}) + \int k(\boldsymbol{x}-\boldsymbol{y}) f(\boldsymbol{y})\,\mathrm{d}\boldsymbol{y}$, where g and k are given functions from weighted Korobov spaces. These spaces are characterized by a smoothness parameter $\alpha > 1$ and weights $\gamma_1 \geq \gamma_2 \geq \cdots$. The weight γ_j moderates the behaviour of the functions with respect to the jth variable. We approximate f by the Nyström method using n rank-1 lattice points. The combination of convolution and lattice group structure means that the resulting linear system can be solved in $O(n \log n)$ operations. We analyze the worst case error measured in sup norm across functions g in the unit ball and a class of functions k in weighted Korobov spaces.

We show that the generating vector of the lattice rule can be constructed component-by-component to achieve the optimal rate of convergence $O(n^{-\alpha/2+\delta})$, $\delta > 0$, with the implied constant independent of the dimension d under an appropriate condition on the weights. This construction makes use of an error criterion similar to the worst case integration error in weighted Korobov spaces, and the computational cost is only $O(n \log n\, d)$ operations.

Keywords: Fredholm integral equation of the second kind, Nystrom method, rank-1 lattice points, Korobov spaces

Joint work of: Josef Dick, Peter Kritzer, Frances Kuo, Ian H. Sloan

Sparsity reconstruction by the standard Tikhonov method

Shuai Lu (Radon Institute for Computational and Applied Mathematics - Linz, A)

It is a common belief that Tikhonov scheme with $\|\cdot\|_{L_2}$-penalty fails in sparsity reconstruction. We are going to show, however, that this standard regularization can help if the stability measured in L_1-norm will be properly taken into account in the choice of the regularization parameter. More precisely, assuming that

$$\|x_\alpha^\delta - x_\alpha^0\|_{L_1} \le c\frac{\delta}{\alpha^\nu}.$$

we use it within the framework of the L_1-balancing principle for parameter choice. Here δ is L_2-data noise level, α is a regularization parameter, and x_α^δ, x_α^0 are Tikhonov regularized solutions corresponding to noisy and pure data respectively. The crucial point is that now a degree of instability ν may depend on the basis with respect to which the solution of the problem is assumed to be sparse. We discuss how this ν can be estimated numerically and present the results of computational experiments giving the evidence of the reliability of our approach.

Keywords: Tikhonov regularization, sparse problems, L_1-balancing problems

Joint work of: Sergei V. Pereverzev, Shuai Lu

Smoothness beyond differentiability

Peter Mathé (Weierstraß Institut, Berlin, D)

We present a recent approach to measuring smoothness of elements in Hilbert space relative to some generating operator. The crucial ingredient will be the notion of an index function. We present results concerning interpolation and approximation in related spaces of smooth functions. Geometric properties of the underlying index functions will turn out to be important.

Keywords: Variable Hilbert scale, interpolation, operator concave functions

Real Computational Universality: The Word Problem for a class of groups with infinite presentation

Klaus Meer (University of Southern Denmark, Odense, DK)

In this talk we introduce a class of groups represented as quotient groups of some free groups generated by infinitely many elements and certain normal subgroups. We show that the related word problem is universal in the Blum-Shub-Smale model of computation, i.e. it has the same difficulty as the real Halting Problem. This is the first natural example of a problem with this property.

Keywords: Computational group theory, word problem, Blum-Shub-Smale model

Joint work of: Klaus Meer, Martin Ziegler

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/877

Quadrature formulas and localized linear polynomial operators on the sphere

Hrushikesh N. Mhaskar (California State University, Los Angeles, USA)

We describe numerical algorithms for construction of quadrature formulas on a Euclidean sphere, exact for spherical polynomials of a high degree. Our formulas are based on scattered sites; i.e., in contrast to such well known formulas as Driscoll–Healy formulas, we need not choose the location of the sites in any particular manner. While the previous attempts to construct such formulas have yielded formulas exact for spherical polynomials of degree at most 18, we are able to construct formulas exact for spherical polynomials of degree 178. The second goal of the paper is to demonstrate the use of these formulas in constructing localized, linear, quasi–interpolatory polynomial operators based on scattered sites. The approximation and localization properties of our operators are studied theoretically in deterministic as well as probabilistic settings. Numerical experiments are presented to demonstrate their superiority over traditional least squares and discrete Fourier projection polynomial approximations.

Keywords: quadrature formulas, localized kernels, polynomial quasi–interpolation, learning theory on the sphere.

Joint work of: Quoc Thong Le Gia, Hrushikesh N. Mhaskar

Complexity of weak approximation of SDEs

Thomas Müller-Gronbach (Universität Magdeburg, D)

Weak approximation of a stochastic differential equation deals with the computation of quantities that only depend on the distribution of the solution X of the equation. In this talk we focus on the computation of expectations $E(f(X))$ for Lipschitz continuous functionals f. We study eterministic as well as randomized (Monte-Carlo) algorithms and we present lower and upper bounds for the minimal worst-case errors. Our analysis exploits the tight connection to the quantization problem for the distribution of X.

Keywords: Stochastic differential equation, weak approximation, Lipschitz functionals

Joint work of: Steffen Dereich, Thomas Müller-Gronbach, Klaus Ritter

Systems of SDEs with additive fractional noise: optimal uniform approximation

Andreas Neuenkirch (Technische Universität Darmstadt, D)

In this talk, we study the approximation of systems of stochastic differential equations with additive fractional noise of any Hurst parameter $H \in (0, 1)$. We derive the exact rate of convergence of the Euler scheme, considering a mean-square L^∞-error criterion. Moreover, we also study the question of optimal approximation schemes.

Keywords: Exact rate of convergence, fractional Brownian motion, lower bounds, optimal discretization, pathwise approximation, stochastic differential equations

Simple Monte Carlo and the Metropolis algorithm

Erich Novak (Universität Jena, D)

We study the integration of functions with respect to a density which is known only up to the normalizing factor. Information is available as oracle calls to the integrand and to the non-normalized weight function. We are interested in analyzing the integration error of optimal algorithms (or the complexity of the problem) with emphasis on the variability of the weight function. For a corresponding large class of problem instances we show that the complexity grows linearly in the variability, and a simple Monte Carlo method provides an almost optimal algorithm. Under additional geometric restrictions (mainly log-concavity) for the class of weight functions, we establish that a suitable adaptive local Metropolis algorithm is almost optimal and much better than any non-adaptive algorithm.

Keywords: Non-linear integration problem, Metropolis algortihm

Joint work of: Peter Mathé, Erich Novak

Lattice sequences in finance

Dirk Nuyens (Katholieke Universiteit Leuven, B)

In a recent paper with Ronald Cools and Frances Kuo we constructed embedded lattice rules with a number of points p^m, which can be used as a low-discrepancy sequency and which are good for all powers of the basis p (a prime).

In this talk we apply the lattice rule included in the appendix of that paper to some financial problems. This lattice rule was optimized for a weighted unanchored Sobolev space with finite order weights of only order 2. In this way there is actually no choice of weights since the weight for the order 1 projections has no influence on the selection.

We show that this specific rule performs very well for two path dependent options:

- an Asian option, for which we use standard, Brownian bridge and PCA path constructions;
- a lookback option, for which we apply a periodizing transform.

We show a convergence graph with the standard error point by point. It can clearly be seen that the constant in the case of periodization is much higher. But also that the standard error can be used to reliably select good points to stop, and which then beat the other low-discrepancy sequences tested.

Keywords: Numerical integration, Quasi-Monte Carlo, lattice rules, fast component-by-component construction, extensible lattice sequences

On the complexity of approximating the minimum eigenvalue of $-\Delta + q$

Anargyros Papageorgiou (Columbia University, USA)

We study the complexity of approximating the minimum eigenvalue of $-\Delta + q$ with Dirichlet boundary condition. We show tight worst case complexity bounds, upper and lower complexity bounds for randomized algorithms and quantum algorithms using bit queries, and tight complexity bounds for quantum algorithms using power queries.

Keywords: Sturm-Liouville problem, worst case complexity, randomized setting, quantum complexity

Adaptive regularization algorithms in learning theory

Sergei V. Pereverzev (Radon Institute for Computational and Applied Mathematics, Linz, A)

We investigate the problem of an adaptive parameter choice for regularization learning algorithms. In the theory of ill-posed problems there is a long history of choosing regularization parameters in optimal way without a priori knowledge of a smoothness of the element of interest. But known parameter choice rules cannot be applied directly in Learning Theory. The point is that these rules are based on the estimation of the stability of regularization algorithms measured in the norm of the space where unknown element of interest should be recovered. But in the context of Learning Theory this norm is determined by an unknown probability measure, and is not accessible.

In the talk we are going to present a new parameter choice strategy consisting in adaptive regularization performed simultaneously in a Hypothesis space and in a space equipped with an empirical norm. Both these spaces are accessible and known parameter choice rules such as a balancing principle can be used there. Then a parameter for the regularization in the inaccessible space is chosen as the minimal among the parameters selected for above mentioned accessible spaces. We prove that under rather mild assumptions such strategy guarantees an optimal order of the risk.

Keywords: Learning theory, regularization theory, adaptive choice of the regularization parameter

Joint work of: Frank Bauer, Sergei V. Pereverzev, Lorenzo Rosasco

The construction of good extensible rank-1 lattices

Friedrich Pillichshammer (University of Linz, A)

It has been shown by Hickernell and Niederreiter that there exist generating vectors for integration lattices which yield small integration errors for $n = p, p^2, \ldots$ for some prime p. In this talk we provide algorithms for the construction of generating vectors which are finitely extensible. The proofs which show that our algorithms yield good extensible rank-1 lattices are based on a sieve principle.

Keywords: Extensible lattice rule, quasi-Monte Carlo, discrepancy

Joint work of: Josef Dick, Friedrich Pillichshammer, Ben J. Waterhouse

Adaptive wavelet schemes for linear parabolic problems

Thorsten Raasch (Philipps-Universität Marburg, D)

We are concerned with the adaptive numerical solution of linear parabolic boundary value problems, formulated as an initial value problem $u'(t) = Au(t) + f(t)$, $u(0) = u_0$ over some Hilbert space. The adaptive schemes under consideration use a linearly implicit semidiscretization in time and adaptive wavelet methods in space for the arising operator equations per time step.

This approach simplifies the convergence and complexity analysis of the overall algorithm. The strong analytical properties of wavelet bases also allow for an efficient preconditioning of the elliptic subproblems. Numerical examples are given to support the theoretical analysis.

Keywords: Linear parabolic b.v. problem, linearly implicit semi discretization, adaptive wavelet approximation

Generalized polynomial lattices with large figure of merit

Wolfgang Ch. Schmid (Universität Salzburg, A)

Dick recently introduced digital $(t, \alpha, \beta, n \times m, s)$-nets, a generalization of digital nets. These nets can also be seen as a generalization of polynomial lattice rules. We will introduce a figure of merit of these polynomial lattices which is related to the quality parameter t and show metrical existence results for the generalized nets. Then we will compare these results to explicit constructions. For our comparisons we will make use of MinT (`http://mint.sbg.ac.at/`), the online database for optimal net parameters. We will end with providing some information on MinT.

Keywords: Digital net, polynomial lattice, figure of merit, MinT

Joint work of: Josef Dick, Peter Kritzer, Friedrich Pillichshammer, Wolfgang Ch. Schmid

Convergence of the coupled cluster method for the electronic Schrödinger equation

Reinhold Schneider (Universität Kiel, D)

The numerical modeling of molecular processes should be based on *first principles* of quantum mechanics. The fundamental *electronic Schrödinger equation*, which explains most phenomena of interest, describes the stationary and non-relativistic behavior of an ensemble of N electron in an electric field caused by fixed the nuclei. Since the corresponding solution depends on at least $3N$ spatial variables a numerical solution of this problem is rather difficult. The *Coupled Cluster Method* (CC-Method) for computing the corresponding wave function has been proved in practice to be an extremely powerful tool. Instead of linear subspace it looks for an approximate solution on a nonlinear sub-manifold. Due to the underlying exponential parametrization it is size consistent. In the present talk we investigate the convergence of the wave function and also of the ground state energy of the numerical solution obtained by the Projected Coupled Cluster Method.

Keywords: Quantum chemistry, electronic Schrödinger equation, projected coupled cluster method

Optimal approximation of elliptic problems by linear and nonlinear mappings

Winfried Sickel (Universität Jena, D)

We study the optimal approximation of the solution of an operator equation $\mathcal{A}(u) = f$ by certain n-term approximations with respect to specific classes of frames. We consider worst case errors, where f is an element of the unit ball of a Sobolev or Besov space $B^t_q(L_p(\Omega))$ and $\Omega \subset I\!R^d$ is a bounded Lipschitz domain; the error is always measured in the H^s-norm. We study the order of convergence of the corresponding nonlinear frame widths and compare it with several other approximation schemes. Our main result is that the approximation order is the same as for the nonlinear widths associated with Riesz bases, the Gelfand widths, and the manifold widths. This order is better than the order of the linear widths iff $p < 2$. The main advantage of frames compared to Riesz bases, which were studied in our earlier papers, is the fact that we can now handle arbitrary bounded Lipschitz domains—also for the upper bounds.

Keywords: n-term approximation, linear operator equation, non linear frame-width

Joint work of: Stephan Dahlke, Erich Novak, Winfried Sickel

Selected nonlinear problems

Krzysztof Sikorski (University of Utah, USA)

We present new complexity results for the fixed point problem in directionally Lipschitz class. Severel numerical tests are included. We also discuss fast algorithms for combustion chemistry fixed point problems, as well as the solution of multivariate polynomial systems originating in the matrix multiplication problem.

Keywords: Somputational complexity, fixed point problems, directionally Lipschitz class

Breaking the curse of dimensionality for integration over product of spheres

Ian H. Sloan (University of New South Wales, AU)

In this paper we construct a QMC integration rule on the product of spheres. In previous work with Frances Kuo we proved non-constructively that, in a certain weighted space setting, there exist QMC rules that have a worst-case error that is bounded independently of dimension. In this present work we show that rules satisfying this property can be constructed

Keywords: Integration on spheres, construction of QMC rules

Joint work of: Kerstin Hesse, Frances Kuo, Ian H. Sloan

Optimality of a standard adaptive finite element method

Rob Stevenson (Utrecht University, NL)

We present a construction an adaptive finite element method for solving elliptic equations in d space dimensions that has optimal computational complexity. Whenever for some $s > 0$, the solution can be approximated to accuracy $\mathcal{O}(N^{-s})$ in energy norm by a continuous piecewise polynomial of degree k on some partition into N d-simplices, and one knows how to approximate the right-hand side in the dual norm with the same rate with piecewise polynomials of degree $k-1$, then the adaptive method produces approximations that converge with this rate, taking a number of operations that is of the order of the number of simplices in the output partition. The method is similar in spirit to that from [*SINUM*, 38 (2000), pp.466–488] by Morin, Nochetto, and Siebert, and so in particular it does not rely on a recurrent coarsening of the partitions.

When time allows, as extensions we discuss a new optimal goal oriented adaptive finite element method, and such a method for solving the Stokes equations, and illustrate our findings with numerical results.

Keywords: Elliptic equations, adapaptive finite element method

Complexity of initial-value problems for ordinary differential equations of order k

Marek Szczęsny (AGH University of Science & Technology, Krakow, PL)

We present results for solving initial-value problems for ordinary differential equations of order k, where the right-hand side function depends on the function of the solution and its derivatives up to q-th order. We answer the question of upper and lower bounds on the complexity for any k and q in the worst-case error model.

We consider two classes of information. We establish matching complexity bounds for standard information. By constructing an algorithm using integral information, we prove that linear information is much more powerful than standard one. We show that in the linear information case upper and lower bounds depend significantly on k (this is not true for standard information).

Keywords: k-th order initial-value problems, standard information, linear information, integral information, worst-case complexity

Discrepancy between QMC and RQMC

Shu Tezuka (Kyushu University, J)

We introduce a class of functions in $d \geq 3$ dimensions which have arbitrary odd effective dimensions in the superposition sense between three and d. We prove that for the integration of any function in this class any Sobol points of a fixed length provide zero-error estimates, whereas Owen's scrambling of any Sobol points of the same length provides the same variance of error as simple Monte Carlo methods.

Keywords: High-dimensional integration, Owen scrambling, QMC, RQMC, Sobol points

Function spaces on unit cube and sampling numbers

Jan Vybíral (Universität Jena, D)

Assume that we want to recover a function $f : \Omega \to \mathbb{C}$ from the unit ball of a Besov space $B^{s_1}_{p_1 q_1}(\Omega)$ by a linear sampling method

$$S_n f = \sum_{j=1}^{n} f(x_j) h_j,$$

in the norm of another Besov space, say $B^{s_2}_{p_2 q_2}(\Omega)$. Here h_j are fixed functions from $B^{s_2}_{p_2 q_2}(\Omega)$, $x_j \in \Omega$ and Ω is a bounded Lipschitz domain in $I\!R^d$.

We prove that the optimal rate of convergence of linear sampling method is

$$n^{-\frac{s_1-s_2}{d}+(\frac{1}{p_1}-\frac{1}{p_2})_+}$$

if $s_2 > 0$ and

$$n^{-\frac{s_1}{d}+(\frac{s_2}{d}+\frac{1}{p_1}-\frac{1}{p_2})_+}$$

if $s_2 < 0$. The same optimal rate is achieved when considering the class of nonlinear sampling methods. In the case $s_2 > 0$ we prove the result only for function spaces on unit cube $\Omega = (0,1)^d$. On the other hand, this allows to describe the sampling operator more explicitly. Finally, we point out, that the result may be simply carried over to the scale of Triebel-Lizorkin spaces, which includes as a special case also Sobolev spaces.

Keywords: Linear and nonlinear approximation methods, Besov and Triebel-Lizorkin spaces, sampling numbers

Non-equidistant time discretization of stochastic heat equations

Tim Wagner (Technische Universität Darmstadt, D)

We consider the non-uniform time discretization for approximation of stochastic heat equations, i.e., one-dimensional components of the driving Wiener process are evaluated at different stepsizes or even non-equidistantly. We show that a proper choice of such a discretization leads to asymptotically optimal algorithms, while asymptotic optimality cannot be achieved by uniform time-discretization, in general.

Keywords: Stochastic heat equation, optimal approximation, non-uniform time discretization

Joint work of: Thomas Müller-Gronbach, Klaus Ritter, Tim Wagner

L_∞-approximation over reproducing kernel Hilbert spaces; worst case setting

Grzegorz Wasilkowski (University of Kentucky, USA)

We consider the worst case complexity of approximating functions form a reproducing kernel Hilbert space with the error measured in the L_∞ norm. Both optimal linear and optimal standard information is considered. In particular, we show that the L_∞ approximation problem in the worst case setting is related to the ρ-weighted L_2 approximation in the average case setting with respect to a Gaussian measure whose covariance function equals the reproducing kernel.

Keywords: Worst case complexity, average case complexity, RKHS, uniform approximation

Construction of extensible Korobov rules

Ben J. Waterhouse (University of New South Wales, AU)

We introduce construction algorithms for Korobov rules for numerical integration which work well for a given set of dimensions simultaneously. The existence of such rules was recently shown by Niederreiter. Here we provide a feasible construction algorithm and an upper bound on the worst-case error in certain reproducing kernel Hilbert spaces for such quadrature rules. The proof is based on a sieve principle recently used by the authors to construct extensible lattice rules. We treat classical lattice rules as well as polynomial lattice rules.

Keywords: Quasi-Monte Carlo methods, (polynomial) lattice rules, Korobov rules.

Joint work of: Josef Dick, Friedrich Pilichshammer, Ben J. Waterhouse

Anisotropic smoothness spaces via level sets

Przemysław Wojtaszczyk (University of Warsaw, PL)

We propose a definition of new smoothness spaces. To define it we first measure "smoothness" of level sets and next we look how "smooth" the level sets are changing. In both cases we measure "smoothness" by the rate of approximation. For those smoothness spaces we compute the rate of approximation.

Keywords: Smoothness spaces, level sets, rate of approximation

Joint work of: Ron DeVore, G. Petrova, Przemyslaw Wojtaszczyk

On generalized tractability for multivariate problems

Henryk Woźniakowski (Columbia University, USA)

We know that linear tensor product problems which are not linear functionals are not polynomially tractable for unweighted Hilbert spaces. We show that a weaker form of tractability holds for such problems, and suggest to study not only polynomial tractability but also generalized tractability for multivariate problems.

Keywords: Generalized tractability, unweighted Hilbert space

Joint work of: Michael Gnewuch, Henryk Woźniakowski