Geometric Distance Estimation for Sensor Networks and Unit Disk Graphs

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Abstract. We present an approach to estimating distances in sensor networks. It works by counting common neighbors, high values indicating closeness. Such distance estimates are needed in many self-localization algorithms. Other than many other approaches, ours does not rely on special equipment in the devices.

1 Introduction

One of the key problems in sensor networks is to let nodes know their location, for example, by storing coordinates w.r.t. a global coordinate system. Unless all nodes are equipped with special localization devices (e.g., GPS/Galileo), there needs to be an algorithm that computes positions based on information available to the network.

Practical localization algorithms often use connectivity information enriched with distance estimates for adjacent nodes [4]. Note that the corresponding decision problem is NP-hard [1]; however, there exist heuristics that are successful in practice.

Various ways to measure distance exist. Examples include the transmission time-of-flight over a wireless channel, the latency of infrared communication, or the strength of a wireless signal that decreases with distance. Good approaches have an average error of about 10–20% of the maximal communication range.

Our approach does not rely on special hardware or node capabilities. Assuming the probability of successful communication decreases with increasing distance, the expected fraction of a node's neighbors that it shares with an adjacent node defines a monotonically decreasing function that can be inverted, resulting in a distance estimator based on this fraction. All that is required is the ability to exchange neighbor lists and a model of communication characteristics.

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2 Distance Estimation

General Case: We assume that nodes are uniformly distributed over the plane, with density δ. That is, the expected number of nodes in a region $A \subset \mathbb{R}^2$ of area $\lambda(A)$ equals $\delta\lambda(A)$. The neighborhood N_i of a node i depends on communication characteristics, which are modelled by an appropriate communication model. We focus on symmetric models only, i.e., $i \in N_j$ iff $j \in N_i$. The model is a probability function $p(d)$ that defines the probability that two nodes i and j with distance $d = \|i - j\|$ are connected. Hence, the expected size of a neighborhood is $\mathbb{E}[|N_i|] = \delta \int_{\mathbb{R}^2} p(\|x\|)dx$ for all nodes i.

We want to estimate the distance of i and j by counting how many of i's neighbors are shared with j. The expected size of this fraction is

$$\begin{aligned} f_p(d) &:= \mathbb{E}[|N_i \cap N_j| / |N_i \setminus \{j\}|] \qquad (1) \\ &= \frac{\int_{\mathbb{R}^2} p(\|x\|)p(\|x-(d,0)^\mathsf{T}\|)dx}{\int_{\mathbb{R}^2} p(\|x\|)dx}, \end{aligned}$$

where $d = \|i - j\|$. If f_p^{-1} exists, two nodes i and j can exchange their neighbor lists, compute the shared fraction $\varphi_{i,j} = |N_i \cap N_j| / |N_i \setminus \{j\}|$ and estimate their distance as $f_p^{-1}(\varphi_{i,j})$. Note that $\varphi_{i,j}$ and $\varphi_{j,i}$ may be different, so some additional tie breaking or averaging scheme must be used.

There is an elegant way to implement this approach for practical purposes, as proposed by Buschmann et al. [2]: Instead of f_p^{-1}, a small discrete value table of f_p is stored in the nodes, and the estimate is done by reverse table lookup. This even works for p or f_p obtained by numerical or field experiments, and it can be implemented using only integer arithmetic.

Scaled density $\pi\delta$	5	8	10	15	20	40	80
Absolute error (inner)	.225	.183	.165	.137	.120	.087	.062
Absolute error (boundary)	.257	.201	.182	.154	.135	.101	.077
Relative error (inner)	.623	.500	.445	.362	.308	.219	.156
Relative error (boundary)	.692	.558	.499	.407	.353	.259	.194

Table 1: Average estimation errors for different densities

Unit Disk Graphs: A widely used model for radio networks is the Unit Disk Graph (UDG), where two nodes i and j are connected by a link iff $\|i - j\| \leqslant 1$.

For UDGs, the estimated neighborhood fraction (1) is $f : [0,1] \to [0,1]$ with

$$f(d) = \frac{2}{\pi}\left(\arctan(\tfrac{d}{2}) - \tfrac{d}{2}\sqrt{1-(\tfrac{d}{2})^2}\right) . \quad (2)$$

The restriction of the domain is feasible because nodes of larger distance than 1 are never neighbors. f^{-1} exists, but unfortunately we lack a closed formula for it. Instead, we approximate f^{-1} by its Taylor series about $f(0) = 1$. Here, we use the 7th-order Taylor polynomial

$$t_7(\varphi) = -\frac{\pi}{1!\,2}(\varphi-1) - \frac{\pi^3}{3!\,2^5}(\varphi-1)^3 - \frac{13\pi^5}{5!\,2^9}(\varphi-1)^5 - \frac{491\pi^7}{7!\,2^{13}}(\varphi-1)^7. \quad (3)$$

We do not use a higher order because evaluating the polynomial on practical embedded systems would become numerically unstable.

There is one issue with this kind of estimation: Real networks do not span the whole plane, but only a certain area A. Nodes that are far from ∂A do not suffer from that, but closer nodes have fewer neighbors. We exploited this fact to actually detect the network boundary [3]. Our distance estimation breaks ties using this effect: When estimating the distance of i and j, the node with a larger neighborhood is likely farther from ∂A than the other, so we use his computed neighborhood fraction. Hence the estimator function we use is

$$\min\{1, t_7(|N_i \cap N_j| / (\max\{|N_i|, |N_j|\} - 1))\}. \quad (4)$$

3 Experiments

To evaluate the UDG estimator's performance, we ran some simulations. Table 1 shows their results. The first row contains the expected size of a neighborhood, without boundary effects. For UDGs, this is $\pi\delta$. Furthermore, the average absolute and relative errors are reported. (For an estimate e for a real distance d, $|e - d|$ is the absolute and $|e - d|/d$ the relative error.) The former equals the error relative to the communication range, which is the common measure for distance estimators. The average is taken separately for two classes of links: For "inner" links, the communication ranges of both end-nodes are fully contained in the network region. For "boundary" ones, both end-nodes lie at most 1 from a straight boundary. This separation has two benefits: First, the estimate in its current form focuses on the inner ones only, and second, it removes the dependency on the network region's shape from the evaluation.

One can see how our approach reaches the desired accuracy of $\leqslant$ 20% at small densities of 10 neighbors at average – for inner nodes. We are conviced that further enhancements are possible by explicitly addressing nodes at the boundary.

References

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