Distributed parameter and state estimation in a network of sensors

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1 Introduction

A wireless sensor network (WSN) consists of spatially distributed autonomous devices using sensors connected via a wireless link. Sensors may be designed for pressure, temperature, sound, vibration, motion... Initially WSN were developed for military applications (battlefield surveillance). Now, many civilian applications (environment monitoring, home automation, traffic control) may take advantage of WSN, see, *e.g.*, [KM04, Hae06].

Applications suggest many research topics, from the design of protocols for communication between sensors, localization problems, data compression and aggregation, security issues... All these problems are made more complicated by the constraints imposed on each node of the WSN, which usually has limited computing capabilities, communication capacity and a very restricted power consumption.

Here, the application we consider is WSN for source tracking, which may be important when considering mobile phone localization and tracking, computer localization in an ad-hoc networks, co-localisation in a team of robots, speaker localization... Figure 1 illustrates a typical localization problem: a source represented by a circle moves in a field of sensors, each of which is represented by a cross.

The localization technique used depends on the type of information available to the sensor nodes. Time of arrival (TOA), time difference of arrival (TDOA) and angle of arrival (AOA) usually provide the best results [PAK+05], however, these quantities are most difficult to obtain, as they require, a good synchronization between timers (for TOA), exchanges between sensors (for TDOA) or multiple antennas (for AOA). Contrary to TOA, TDOA or AOA data, readings of signal strength (RSS) at a given sensor are easily obtained, as they only require low-cost sensors or are already available, as in IEEE 802.11 wireless networks, where these data are provided by the MAC layer [STK05].

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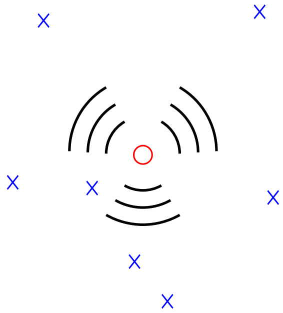
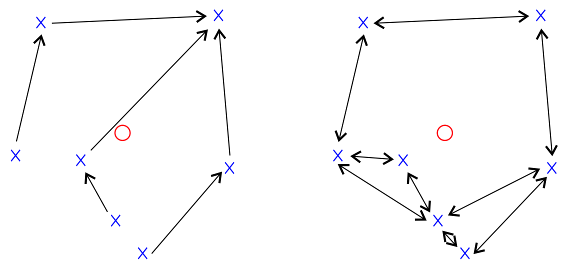


Figure 1: Source (o) and sensors (x)

This paper focuses on source localization from RSS data. Centralized approaches (see Figure 2, left) have been proposed to solve this problem for acoustic sources [SH05] and for sources emitting electromagnetic waves, see, *e.g.*, [KMR$^+$04, GG05, GTG$^+$05]. In the first case, some knowledge of the decay rate of the RSS (*path loss exponent*) is needed for efficient nonlinear least squares estimation. In the second case, an off-line training phase is required to allow maximum *a posteriori* localization. In both cases, a good initial guess of the location of the source facilitates convergence to the global minimum of the cost function. Distributed approaches (see Figure 2, right) have also been employed, *e.g.*, in [RN04], where a distributed version of nonlinear least squares has been presented. When badly initialized, it suffers from the same convergence problems as the centralized approach, as illustrated in [HIB05], which advocates projection on convex sets. However, this requires an accurate knowledge of the source signal strength and of the path loss exponent.



Centralized Distributed

Figure 2: Centralized (left) and distributed (right) processing of measurements

Here, we consider a distributed state estimation algorithm involving bounded measurement errors. This problem will be addressed with the help of interval analysis, which will provide at each network node a set estimate guaranteed to contain the true location of a moving source, provided the hypotheses on the model and measurement noise are satisfied.

2 Distributed state estimation

Consider a system described by a discrete-time dynamical model

$$\mathbf{x}_k = \mathbf{f}_k\left(\mathbf{x}_{k-1}, \mathbf{w}_k, \mathbf{u}_k\right), \tag{1}$$

where $\mathbf{x}_k$ is the state vector of the model at time instant k (the sampling period is T). The state perturbation vector $\mathbf{w}_k$ accounts for unmodelled parts of the system. This noise vector is assumed to remain in a known box $[\mathbf{w}]$. The input vector $\mathbf{u}_k$ is also assumed known. At $k = 0$, $\mathbf{x}_0$ is only assumed to belong to some (possibly large) known set $\mathbb{X}_0$.

Assume that at time k, each sensor $\ell = 1 \dots L$ of a WSN has access to a measurement

$$\mathbf{y}_k^\ell = \mathbf{g}_k^\ell\left(\mathbf{x}_k, \mathbf{v}_k^\ell\right), \tag{2}$$

where $\mathbf{y}_k^\ell$ is the noisy measurement vector, and $\mathbf{v}_k^\ell$ is the measurement noise, assumed bounded in some known $[\mathbf{v}]$. Then (1) and (2) are the dynamic and observation equations of the model. Usual measurement equations are

$$\mathbf{g}_k^\ell\left(\mathbf{x}_k, \mathbf{v}_k^\ell\right) = \mathbf{h}_k^\ell\left(\mathbf{x}_k\right) + \mathbf{v}_k^\ell \tag{3}$$

or

$$\mathbf{g}_k^\ell\left(\mathbf{x}_k, v_k^\ell\right) = \mathbf{h}_k^\ell\left(\mathbf{x}_k\right).v_k^\ell. \tag{4}$$

2.1 Back to centralized discrete-time state estimation

When all measurements at time k are available at central processing unit, one gets

$$\begin{cases} \mathbf{x}_k = \mathbf{f}_k\left(\mathbf{x}_{k-1}, \mathbf{w}_k, \mathbf{u}_k\right), \\ \mathbf{y}_k = \mathbf{g}_k\left(\mathbf{x}_k, \mathbf{v}_k\right), \end{cases} \tag{5}$$

with $\mathbf{y}_k^{\mathrm{T}} = \left(\left(\mathbf{y}_k^1\right)^{\mathrm{T}}, \dots, \left(\mathbf{y}_k^L\right)^{\mathrm{T}}\right)$ and $\mathbf{v}_k^{\mathrm{T}} = \left(\left(\mathbf{v}_k^1\right)^{\mathrm{T}}, \dots, \left(\mathbf{v}_k^L\right)^{\mathrm{T}}\right)$. Determining an estimate for $\mathbf{x}_k$ from the measurement $\mathbf{y}_\ell$, $\ell = 0 \dots k$ is a classical state estimation problem, which solution depends on linearity of (1) and (2) and noise model. For a gaussian noise, with a linear dynamic equation, the Kalman filter [Kal60] is the natural solution. When the model is non-linear, one may use an extended Kalman filter [Gel74], griddling techniques [TA95] or particle filters [PS99]. In a bounded-error context, with a linear model, the set of state vectors consistent with the model and noise on the measurements may be evaluated exactly using polytopes [Sch68], or outer-approximated using ellipsoids [MN96]. With a nonlinear model, again, only an outer-approximation of the state is possible using subpavings, *i.e.*, union of non-overlapping boxes [KJW02].

Summarizing the information available at time k, one gets

$$\mathcal{I}_k = \left\{\mathbb{X}_0, \left\{[\mathbf{w}_j]\right\}_{j=1}^k, \left\{[\mathbf{v}_j]\right\}_{j=1}^k, \left\{[\mathbf{y}_j]\right\}_{j=1}^k\right\}. \tag{6}$$

Centralized bounded-error state estimate at time k aims at characterizing the set $\mathbb{X}_{k|k}$ of all values of $\mathbf{x}_k$ that are consistent with (1), (2) and $\mathcal{I}_k$. One may propose an idealized algorithm [KJW02], alternating, as the Kalman filter a prediction step

$$\mathbb{X}_{k|k-1} = \left\{ \mathbf{f}_k \left(\mathbf{x}, \mathbf{w}, \mathbf{u}_k \right) \ | \ \mathbf{x} \in \mathbb{X}_{k-1|k-1}, \ \mathbf{w} \in [\mathbf{w}] \right\} \tag{7}$$

and a correction step accounting for the new measurement

$$\mathbb{X}_{k|k} = \left\{ \mathbf{x} \in \mathbb{X}_{k|k-1} \mid \mathbf{y}_k = \mathbf{g}_k \left(\mathbf{x}, \mathbf{v} \right), \ \mathbf{v} \in [\mathbf{v}]^{\times L} \right\}. \tag{8}$$

The two steps of the idealized algorithm are depicted in Figure 3.

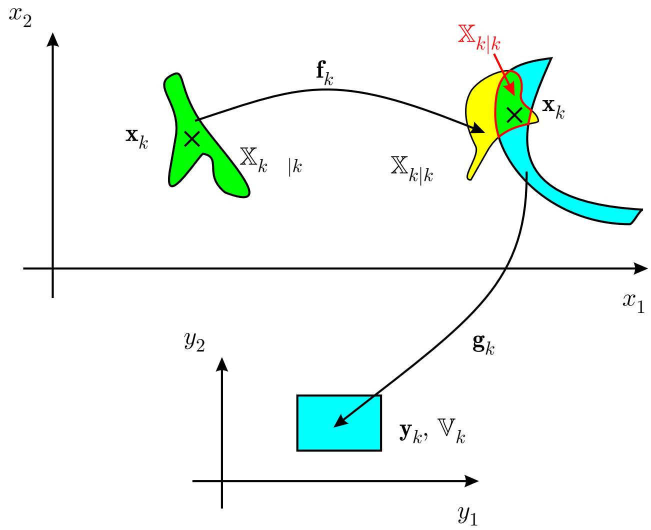


Figure 3: Idealized recursive bounded-error state estimator

2.2 Distributed state estimation

Ideally, any sensor ℓ, $\ell = 1 \ldots L$ of the WSN should provide

$$\mathbb{X}^{\ell}_{k|k} = \mathbb{X}_{k|k}. \tag{9}$$

Distributed versions of the Kalman filter have been proposed in [Spe79], assuming linear models, gaussian noise, and instantaneous communications. Application to distributed estimation in power systems have been addressed in [LC05] and to distributed estimation in WSN are considered in [RGR08]. Nevertheless, to the best of our knowledge, no similar tools have been proposed in a bounded-error context.

2.2.1 Hypotheses

Here, we assume that the sensor network is entirely connected, which is a necessary condition to be able to obtain $\mathbb{X}^{\ell}_{k|k} = \mathbb{X}_{k|k}$. The following measurement processing and communication will be considered. At time k, each sensor processes own measurement $\mathbf{y}_k^\ell$. Between time k and $k+1$, a first round trip is considered ($r=1$) in which each sensor ℓ broadcasts its own estimate $\mathbb{X}_{k|k}^{\ell,r}$ to the network. Then each sensor ℓ receives and processes $\mathbb{X}_{k|k}^{s,1}$, $s \in \mathcal{C}(\ell)$ where $\mathcal{C}(\ell)$ is the set of indices of sensors connected to ℓ. Depending on the sample time, more round trips ($r>1$) may be considered. Just before time $k+1$, each sensor ℓ builds a final estimate $\mathbb{X}_{k|k}^\ell$.

This way of processing and transmitting information leads to the following idealized distributed algorithm.

2.2.2 Proposed idealized algorithm

For each sensor $\ell = 1 \dots L$,

At time k:

$$\mathbb{X}_{k|k-1}^\ell = \left\{ \mathbf{f}_k \left(\mathbf{x}, \mathbf{w}, \mathbf{u}_k \right) \mid \mathbf{x} \in \mathbb{X}_{k-1|k-1}^\ell,\ \mathbf{w} \in [\mathbf{w}] \right\}. \tag{10}$$

$$\mathbb{X}_{k|k}^{\ell,0} = \left\{ \mathbf{x} \in \mathbb{X}_{k|k-1}^\ell \mid \mathbf{y}_k^\ell = \mathbf{g}_k^\ell \left(\mathbf{x}, \mathbf{v} \right),\ \mathbf{v} \in [\mathbf{v}] \right\}. \tag{11}$$

Between k and $k+1$,
for $r = 1$ to $R_{\max}$ (number of round trips)

$$\mathbb{X}_{k|k}^{\ell,r} = \bigcap_{s \in \mathcal{C}(\ell)} \mathbb{X}_{k|k}^{s,,r-1} \tag{12}$$

Just before $k+1$

$$\mathbb{X}_{k|k}^\ell = \mathbb{X}_{k|k}^{\ell,R_{\max}}. \tag{13}$$

One may easily prove that

$$\mathbb{X}_{k|k} \subset \mathbb{X}_{k|k}^\ell. \tag{14}$$

However, the conditions to have

$$\mathbb{X}_{k|k} = \mathbb{X}_{k|k}^\ell \tag{15}$$

needs further developments. They involve network connectivity, which itself is linked to the largest distance between sensors. This problem has been partly addressed in [Yok01, BFV$^+$05].

2.2.3 Practical algorithm

The implementation of the proposed idealized algorithm is done in a way similar to that of the centralized algorithm presented in [KJW02]. In a most basic version of the algorithm, sets are represented by boxes, basic interval evaluations are performed for the prediction step and interval constraint propagation is done for the correction step. The advantage of this version is that it may readily be implemented on chips with reduced computational capabilities. A more sophisticated version involves description of sets using subpavings, a prediction step implemented using ImageSp [KJW02]and Sivia [JW93] combined with interval constraint propagation for the correction step.

3 Applications

For the application part, a static localization problem for a single source is considered first. Then, the source will be moving, and the localization problem is cast into a problem of state estimation.

3.1 Static source localization

The known location of the sensors is denoted by $\mathbf{r}_\ell \in \mathbb{R}^2$, $\ell = 1 \ldots L$. The unknown location of the source is $\boldsymbol{\theta} = (\theta_1, \theta_2)^{\mathrm{T}} \in \mathbb{R}^2$. The mean power $\overline{P}\,(d_\ell)$ (in dBm) received by ℓ-th sensor is described by Okumura-Hata model [OOKF68]

$$\overline{P}_{\mathrm{dB}}\,(d_\ell) = P_0 - 10 n_{\mathrm{p}} \log \frac{d_\ell}{d_0}, \tag{16}$$

where n_{p} is the path-loss exponent (unknown, but constant), $d_\ell = |\mathbf{r}_\ell - \boldsymbol{\theta}|$. The received power is assumed to lie within some bounds

$$P_{\mathrm{dB}}\,(d) \in \left[P_0 - 10 n_{\mathrm{p}} \log \frac{d}{d_0} - e, P_0 - 10 n_{\mathrm{p}} \log \frac{d}{d_0} + e\right], \tag{17}$$

where e is assumed known.

The RSS by sensor $\ell = 1 \ldots L$ may be rewritten as

$$y_\ell = h_\ell\,(\boldsymbol{\theta}, A, n_{\mathrm{p}})\, v_\ell, \tag{18}$$

with

$$h_\ell\,(\boldsymbol{\theta}, A, n_{\mathrm{p}}) = \frac{A}{|\mathbf{r}_\ell - \boldsymbol{\theta}|^{n_{\mathrm{p}}}},\ A = 10^{P_0/10} d_0^{n_{\mathrm{p}}}, \tag{19}$$

and

$$v_\ell \in [v] = \left[10^{-e/10}, 10^{e/10}\right]. \tag{20}$$

The noise is thus multiplicative in the normal domain. The parameter vector to be estimated is then $\mathbf{x} = (A, n_{\mathrm{p}}, \theta_1, \theta_2)^{\mathrm{T}}$.

3.1.1 Distributed approach: interval constraint propagation

At sensor ℓ, $y_\ell \in [y_\ell]$ is measured. Some boxes $[\boldsymbol{\theta}]$, $[A]$, and $[n_{\mathrm{p}}]$ are assumed to be available, *a priori*, or as results transmitted by the other sensors to sensor ℓ. The parameter vector has to satisfy the constraint provided by RSS model

$$y_\ell - \frac{A}{|\mathbf{r}_\ell - \boldsymbol{\theta}|^{n_{\mathrm{p}}}} = 0. \tag{21}$$

Sensor	68	741	954
Measurement	$[9.303, 58.698]$	$[17.856, 112.664]$	$[18.644, 117.640]$

Table 1: Static localization example of measurements

Using interval constraint propagation, it is possible to reduce the domains for the variables using (21). The contracted domains may be written as

$$\begin{cases} [y'_\ell] = [y_\ell] \cap \dfrac{[A]}{|\mathbf{r}_\ell - [\boldsymbol{\theta}]|^{[n_\mathrm{p}]}}, \\ [A'] = [A] \cap [y'_\ell] \, |\mathbf{r}_\ell - [\boldsymbol{\theta}]|^{[n_\mathrm{p}]}, \\ [n'_\mathrm{p}] = [n_\mathrm{p}] \cap \left(\log\left([A']\right) - \log\left([y'_\ell]\right)\right) / \log\left(|\mathbf{r}_\ell - [\boldsymbol{\theta}]|\right), \\ [\theta'_1] = [\theta_1] \cap \left(r_{\ell,1} \pm \sqrt{\left([A'] \,/\, [y'_\ell]\right)^{2/[n'_\mathrm{p}]} - \left(r_{\ell,2} - [\theta_2]\right)^2}\right), \\ [\theta'_2] = [\theta_2] \cap \left(r_{\ell,2} \pm \sqrt{\left([A'] \,/\, [y'_\ell]\right)^{2/[n'_\mathrm{p}]} - \left(r_{\ell,1} - [\theta_1]\right)^2}\right). \end{cases} \quad (22)$$

3.1.2 Simulation results

A networks of $L = 2000$ sensors randomly distributed over a field of 100 m×100 m is considered. The source is placed at $\boldsymbol{\theta}^* = (50\text{ m}, 50\text{ m})$ and emits a wave with $P_0 = 20$ dBm, $d_0 = 1$ m. The path-loss exponent $n_\mathrm{p} = 2$ is assumed to be constant over the field. The measurement noise such that $e = 4$ dBm. Table 1 provides some examples of the measurements which are available to the sensors.

For 100 realizations of the sensor field, data have been simulated with (17). To limit computational load, only sensors such that $y_\ell > 10$ participate to localization. The initial search box for $\mathbf{p}$ is taken as $[0, 100] \times [0, 100] \times [50, 200] \times [2, 4]$ in a first scenario, where A (or P_0) is assumed unknown. In a second scenario, A is assumed perfectly known. For the distributed approach, five cycles in the sensor network are performed.

The two proposed techniques are compared to localization by a closest point approach (CPA), which searches for the index of the sensor with the largest RSS $\ell_{\mathrm{CPA}} = \arg\max_\ell y_\ell$ and uses the location of this sensor $\widehat{\boldsymbol{\theta}}_{\mathrm{CPA}} = \mathbf{r}_{\ell_{\mathrm{CPA}}}$ as an estimate for $\boldsymbol{\theta}^*$. This technique, albeit it is not the most efficient [SH05], performs well for dense sensor networks, as here. Point estimates for $\boldsymbol{\theta}^*$ are evaluated as $\widehat{\boldsymbol{\theta}}_\mathrm{C} = \mathrm{mid}\big(\big[\mathrm{proj}_{\boldsymbol{\theta}}\overline{\mathbb{P}}\big]\big)$, the midpoint of the smallest box containing the projection of $\overline{\mathbb{P}}$ onto the $\boldsymbol{\theta}$-plane in the centralized approach and as the center of the projection onto the $\boldsymbol{\theta}$-plane of the solution box $[\mathbf{p}]$, $\widehat{\boldsymbol{\theta}}_\mathrm{D} = \mathrm{mid}(\mathrm{proj}_{\boldsymbol{\theta}}\,[\mathbf{p}])$, in the distributed approach.

Figures 4 and 5 provides typical solutions obtained using a centralized and distributed localization algorithm. The centralized algorithm involves set description using subpavings, whereas the distributed one only uses boxes.

Figure 6 presents the histogram of the L_2 norm of the difference between $\boldsymbol{\theta}^*$ and its estimates provided by the three techniques previously described.

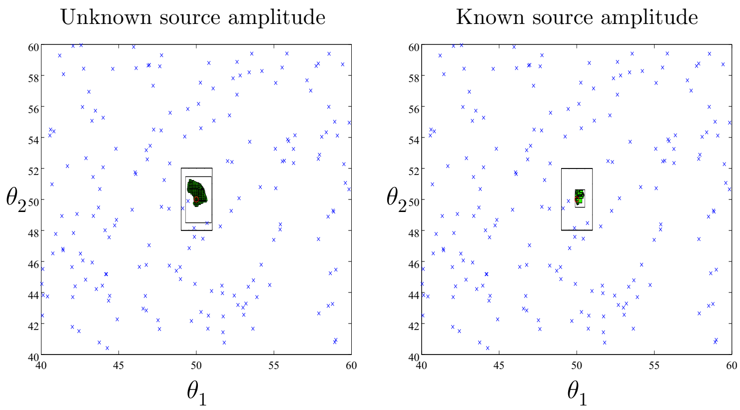


Figure 4: Projection of the solution on the (θ_1, θ_2)-plane

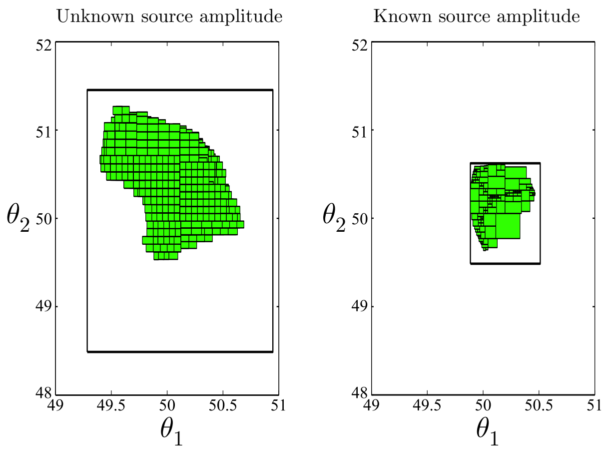


Figure 5: Zoom of the projection of the solution on the (θ_1, θ_2)-plane

The centralized approach performs better than the distributed one, but the distributed approach provides a reasonable estimate at a much lower computation and transmission cost. Both techniques outperforms CPA, the performances of which do not depend on whether A is known.

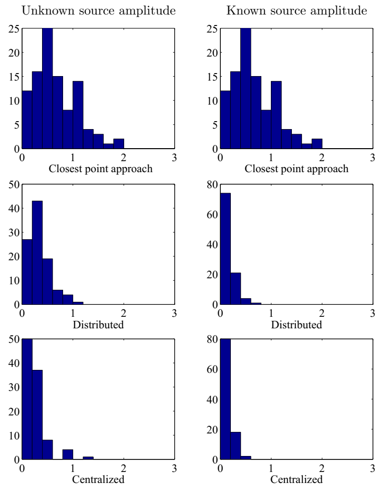


Figure 6: Histograms of estimation error for $\boldsymbol{\theta}$ (100 realizations of sensor field

3.2 Source tracking

In this part, the source is assumed to be moving. A and n_{p} are now known. The state vector is taken as

$$\mathbf{x}_k = (\theta_{1,k}, \theta_{2,k}, \phi_{1,k}, \phi_{2,k}, \theta_{1,k-1}, \theta_{2,k-1}, \phi_{1,k-1}, \phi_{2,k-1})^{\mathrm{T}} \tag{23}$$

where (ϕ_1, ϕ_2) represents the speed with respect to (θ_1, θ_2). This long state vector allows to estimate $\left(\phi_{1,k}, \phi_{2,k}\right)$.

3.2.1 Model

The following uncertain linear dynamic equation is considered to determine the evolution with time of $\mathbf{x}_k$

$$\begin{pmatrix} \theta_{1,k} \\ \theta_{2,k} \\ \phi_{1,k} \\ \phi_{2,k} \\ \theta_{1,k-1} \\ \theta_{2,k-1} \\ \phi_{1,k-1} \\ \phi_{2,k-1} \end{pmatrix} = \begin{pmatrix} \mathbf{I}_4 & \mathbf{0}_4 \\ \mathbf{I}_4 & \mathbf{0}_4 \end{pmatrix} \begin{pmatrix} \theta_{1,k-1} \\ \theta_{2,k-1} \\ \phi_{1,k-1} \\ \phi_{2,k-1} \\ \theta_{1,k-2} \\ \theta_{2,k-2} \\ \phi_{1,k-2} \\ \phi_{2,k-2} \end{pmatrix} + T. \begin{pmatrix} \phi_{1,k-1} \\ \phi_{2,k-1} \\ w_1 \\ w_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{24}$$

with $w_1 \in [w]$ and $w_2 \in [w]$.

3.2.2 Interval constraint propagation

Using interval constraint propagation, one gets the contracted domains

$$\begin{cases} \left[y'_{\ell,k}\right] = [y_{\ell,k}] \cap \dfrac{A}{|\mathbf{r}_\ell - [\boldsymbol{\theta}_k]|^{[n_\mathrm{p}]}}, \\ [\theta'_{1,k}] = [\theta_{1,k}] \cap \left(r_{\ell,1} \pm \sqrt{\left(A/\left[y'_{\ell,k}\right]\right)^{2/n_\mathrm{p}} - (r_{\ell,2} - [\theta_{2,k}])^2} \right), \\ [\theta'_{2,k}] = [\theta_{2,k}] \cap \left(r_{\ell,2} \pm \sqrt{\left(A/\left[y'_{\ell,k}\right]\right)^{2/n_\mathrm{p}} - (r_{\ell,1} - [\theta_{1,k}])^2} \right). \\ [\phi'_{1,k}] = [\phi_{1,k}] \cap \left(\dfrac{[\theta'_{1,k}] - [\theta'_{1,k}]}{T} + T\,[w] \right) \\ [\phi'_{2,k}] = [\phi_{2,k}] \cap \left(\dfrac{[\theta'_{2,k}] - [\theta'_{2,k}]}{T} + T\,[w] \right) \end{cases} \quad (25)$$

3.2.3 Results

Now, a field of 50 m×50 m is considered, with its origin at center. A WSN of $L = 25$ sensors with communication range of 15 m is spread over this field. The source is placed at $\boldsymbol{\theta}^* = (5\text{ m}, 5\text{ m})$, with characteristics $\overline{P}_0 = 20$ dBm, $d_0 = 1$ m. The measurement noise is such that $e = 4$ dBm. The path-loss exponent is $n_\mathrm{p} = 2$, assumed constant over the field. The sampling time is $T = 0.5$ s and $[\mathbf{w}] = [-0.5, 0.5]^{\times 2}$ m.s^{-2}.

Figure 7 illustrates the connectivity of the considered random WSN and the trajectory followed by the source.

The simplest algorithm implementation presented in Section 2.2.3 has been considered: sets are represented by boxes, simple image evaluations using inclusion functions are performed and correction is done by interval constraint propagation. The localization performance using this algorithm is depicted in Figure 8. The width of the solution box (left part of Figure 8) provided at each time instant decreases very quickly to reach a floor. A similar behavior is seen for the norm of the localization taking the center of the solution boxes at each time instant. The convergence is quite fast and the number of round trips has only a very limited impact on the convergence of the algorithm.

4 Conclusions

In this paper, we have considered distributed bounded-error state estimation applied to the problem of source tracking with a network of wireless sensors. Estimation is performed in a distributed context, *i.e.*, each sensor has only a limited amount of measurements available. A guaranteed set estimator is put at work

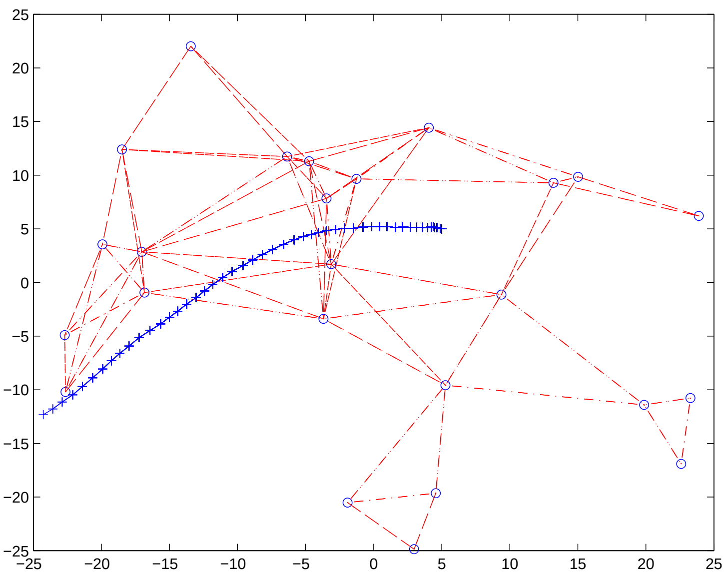


Figure 7: Trajectory of the source (o) in the WSN, each sensor is represented by a cross (x)

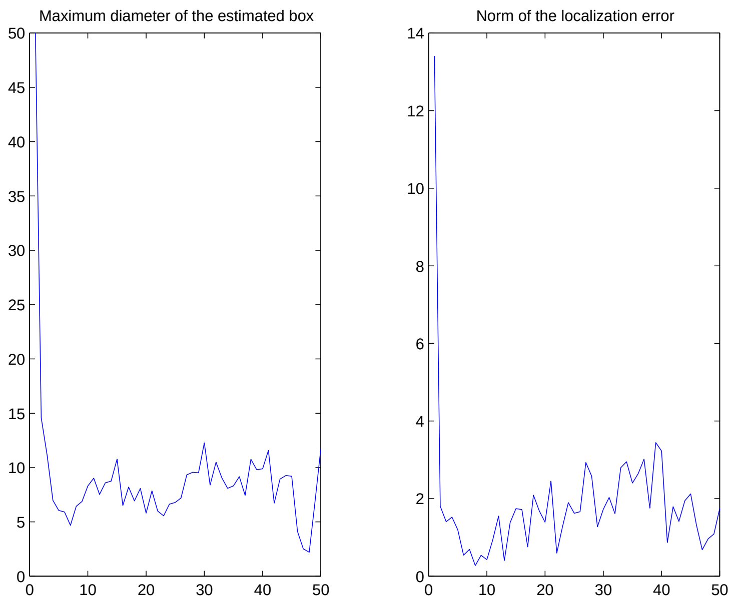


Figure 8: Width of the box $[\theta_{1,k}] \times [\theta_{2,k}]$, and norm of the localization error when the estimate is taken as the center of the solution box

There is still large space for improvements in the considered problem. First, convergence properties have to be carefully studied. In particular, conditions under which the distributed solution coincides with the centralized one have to be determined. Robustness to outliers has also to be considered. A challenging future application would be the distributed estimation, *e.g.*, in a team of robots.

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