# 08051 Abstracts Collection Theory of Evolutionary Algorithms

— Dagstuhl Seminar —

Dirk V. Arnold<sup>1</sup>, Anne Auger<sup>2</sup>, Carsten Witt<sup>3</sup> and Jonathan E. Rowe<sup>4</sup>

Dalhousie University, CA dirk@cs.dal.ca
INRIA Futurs - Paris, FR anne.auger@lri.fr
Universität Dortmund, D cw01@ls2.cs.uni-dortmund.de
University of Birmingham, GB
j.e.rowe@cs.bham.ac.uk

Abstract. From Jan. 27, 2008 to Feb. 1, 2008, the Dagstuhl Seminar 08051 "Theory of Evolutionary Algorithms" was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

**Keywords.** Evolutionary Computation, Theory of Evolutionary Algorithms

### 08051 Executive Summary – Theory of Evolutionary Algorithms

The 2008 Dagstuhl Seminar "Theory of Evolutionary Algorithms" was the fifth in a firmly established series of biannual events. In the week from Jan. 27, 2008 to Feb. 1, 2008, 47 researchers from nine countries discussed their recent work and trends in evolutionary computation. Abstracts of the presentations are put together in the Proceedings. Links to extended abstracts or full papers are provided if available.

Keywords: Evolutionary Algorithms, Theory of Evolutionary Algorithms

Joint work of: Arnold, Dirk , Auger, Anne , Rowe, Jon , Witt, Carsten

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1481

#### Step Length Adaptation on Ridge Functions

Dirk V. Arnold (Dalhousie University, CA)

Step length adaptation is central to evolutionary algorithms in real-valued search spaces. This talk contrasts several step length adaptation algorithms for evolution strategies on a family of ridge functions. The algorithms considered are cumulative step length adaptation, a variant of mutative self-adaptation, twopoint adaptation, and hierarchically organised strategies. In all cases, analytical results are derived that yield insights into scaling properties of the algorithms.

Keywords: Evolution strategies, step length adaptation, ridge functions

Joint work of: Arnold, Dirk V.; MacLeod, Alex

#### Kolmogorov Complexity and Hardness

Yossi Borenstein (University of Essex, GB)

The Kolmogorov complexity (KC) of a string is defined as the length of the shortest program that can print that string and halts. There are several works which try relate this notion to problem hardness in the black box scenario. In this presentation some initial steps are made towards a rigorous run time analysis of the (1+1) EA over incompressible fitness functions.

I am looking forward to getting some feedback and suggestions (towards the next possible steps).

Runtime analysis, Kolmogorov Complexity, Evolutionary Algo-Keywords: rithms

#### Evolutionary Multi-objective Worst Case Optimization

Jürgen Branke (Universität Karlsruhe, D)

Many real-world optimization problems are subject to uncertainty. A possible goal is then to find a solution which is robust in the sense that it has the best worst-case performance over all possible scenarios. However, if the problem also involves multiple objectives, what is "best" or "worst" depends on the user's weighting of the different criteria, which is generally difficult to specify before alternatives are known. Evolutionary multi-objective optimization avoids this problem by searching for the whole front of Pareto optimal solutions. This paper extends the concept of Pareto dominance to worst case optimization problems and demonstrates how evolutionary algorithms can be used for worst case optimization in a multi objective setting.

Keywords: Evolutionary algorithm, worst-case, robustness

#### A Non-artificial Problem Where Crossover Provably Helps

Benjamin Doerr (MPI für Informatik - Saarbrücken, D)

We discuss how to solve the classical all-pairs-shortest-path problem via simple evolutionary/genetic algorithms. Our rigorous run-time analysis shows that the use of crossover does speed-up the optimization time. This is the first time that such a behaviour is proven for a non-artificial problem.

Keywords: Runtime analysis, graph algorithms, crossover, paths in graphs

Joint work of: Doerr, Benjamin; Happ, Edda; Klein, Christian

#### NP-hard Cases of Optimal Recombination

Anton Eremeev (Sobolev Institute of Mathematics - Omsk, RUS)

A number of promising hybrid genetic algorithms are based on the optimized crossover operators that aim at finding the best possible offspring as a result of a recombination of two parent solutions. To evaluate the applicability of this approach, in this talk we consider the complexity of optimal recombination problem in Boolean Programming. We show that the optimal recombination problem for the linear Boolean programming problems with at most two variables per inequality is efficiently solvable.

Also we identify several NP-hard cases of optimal recombination, including the one-dimensional knapsack and the p-median problem.

#### Performance of Evolution Strategies on PDQFs

Steffen Finck (Fachhochschule Vorarlberg, A)

Summary about the research on the behavior of Evolution Strategies with positive definite quadratic forms (PDQFs) as fitness function. First the mutative self-adaptation ( $\sigma$ SA) is analyzed with focus on self-adaptation response and progress rates. For the latter, an optimal setting of the learning parameter is derived. If the learning parameter differs from the optimal setting mutative self-adaptation fails. Second the influence of noise on PDQFs is investigated. Formulae for progress rates and steady state distances are derived and compared to simulations. Additionally the equipartition conjecture is used to derive the steady state distances in the case of vanishing mutation strength. At last, the use of weighted recombination on PDQFs is theoretically analyzed and formulae for optimal weights are presented.

Keywords: Evolution Strategy, Self-Adaptation, Weighted Recombination, Noise

#### Evolution strategies with Random Numbers of Offspring

Olivier Francois (TIMC Laboratory, F)

Evolution strategies rely on both stochastic and deterministic internal parameter settings. For example the step length parameter is usually adapted stochastically, whereas static deterministic values are often chosen for the number of parents, mu, and for the number of offspring, lambda. In this talk, I present a model in which the number of offspring in each generation, lambda, is randomly sampled according to the Poisson distribution. Assuming that the mean of the Poisson distribution is small, i describe the average hitting time of a global optimum for any discrete function defined on an arbitrary finite lattice. The result is based on an analogy with an algorithm for which individuals evolve either under mutation or rank-based selection, but the two operators are never applied simultaneously to a given parent in a given generation.

#### Toward a Convergence Proof for CMA-ES—and Beyond

Nikolaus Hansen (INRIA Futurs - Orsay, F)

Based on the theory of  $\varphi$ -irreducible Markov chains, a convergence proof for the CMA-ES (Covariance Matrix Adaptation Evolution Strategy) is sketched. In the first step, the original algorithm is reduced to the rank-one update of the covariance matrix. A suitable Markov chain is derived and convergence of the chain to a stationary limit distribution, the invariant measure, is shown on a class of unimodal objective functions. The result implies log-linear convergence to the optimum, or log-linear divergence. Even though the simplified version of the algorithm is not recommended in real applications, its convergence is highly relevant. Additional components have only been expected to improve convergence behavior. In further steps a less simplified algorithm and EDAs will be tackled.

Keywords: Evolution strategies, covariance matrix adaptation, CMA, convergence proof

Joint work of: Hansen, Nikolaus; Auger, Anne

#### A Comparison of GAs Penalizing Infeasible Solutions and Repairing Infeasible Solutions on the 0-1 Knapsack Problem

Jun He (University of Birmingham, GB)

Constraints exist in almost every optimization problem. Different constraint handling techniques have been incorporated with genetic algorithms (GAs), however most of current studies are based on computer experiments.

An example is Michalewicz's comparison among GAs using different constraint handling techniques on the 0-1 knapsack problem. The following phenomena are observed in experiments: 1) the penalty method needs more generations to find a feasible solution to the restrictive capacity knapsack than the repair method; 2) the penalty method can find better solutions to the average capacity knapsack. Such observations need a theoretical explanation. This paper aims at providing a theoretical analysis of Michalewicz's experiments. The main result of the paper is that GAs using the repair method are more efficient than GAs using the penalty method on both restrictive capacity and average capacity knapsack problems. This result of the average capacity is a little different from Michalewicz's experimental results. So a supplemental experiment is implemented to support the theoretical claim. The results confirm the general principle pointed out by Coello: a better constraint-handling approach should tend to exploit specific domain knowledge.

Keywords: Genetic Algorithms, Constrained Optimization, Knapsack Problem, Computation Time, Performance Analysis

Joint work of: He, Jun; Zhou, Yuren; Yao, Xin

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1482

#### Automated Tuning, Configuration and Synthesis of Complex Stochastic Local Search Algorithms

Holger H. Hoos (University of British Columbia - Vancouver, CA)

Stochastic local search (SLS) algorithms, like many other heuristic methods for solving hard computational problems, are often built in an incremental way, guided by the experience and intuition of the algorithm designer. This is particularly the case of relatively complex SLS methods, such as evolutionary algorithms or hybrid approaches that combine various simpler search strategies.

As a result, effective SLS strategies for many academic and real-world problems often consist of many interacting components, whose behaviour and interaction is controlled by numerous parameters.

In this talk, I will discuss strategies for automatically configuring these components and for tuning their parameters in order to better realise the overall performance potential of the complex algorithm. I will describe ParamILS, a recent, SLS-based algorithm configuration procedure developed in my group, that we have applied very successfully to a number of high-performing heuristic search methods, ranging from SLS algorithms to SAT to the widely used industrial mixed-integer-programming solver CPLEX. I will also review a formal model of hybrid SLS algorithms, Generalised Local Search Machines, and explain how, in conjunction with automated configuration methods, this model can provide the basis for the principled and automated synthesis of effective hybrid SLS algorithms.

Keywords: Stochastic local search, configuration, tuning, synthesis, hybrid algorithms, meta-algorithmics, heuristics

## Efficient Covariance Matrix Update for Evolution Strategies

Christian Igel (Ruhr-Universität Bochum, D)

Evolution Strategies (ESs) usually rely on normally distributed additive variations of candidate solutions. Adaptation of the covariance matrix of the normal distribution allows efficient search in non-separable and ill-conditioned land-scapes. To sample the distribution, the adapted covariance matrix needs to be decomposed. We propose a new update mechanism that can equivalently replace a rank-one covariance matrix update including the computationally expensive decomposition of the covariance matrix. The new update rule reduces the computational complexity from  $O(n^3)$  to  $O(n^2)$ , where n is the search space dimension.

Keywords: Variable metric algorithm, evolution strategy, covariance matrix adaptation, Cholesky CMA

Joint work of: Suttorp, Thorsten; Hansen, Nikolaus; Igel, Christian

### Bit Flip Mutations vs. Local Search: An Open Problem Presentation

Thomas Jansen (Technische Universität Dortmund, D)

Evolutionary algorithms with binary encoding often employ standard bit flip mutations as variation operator. In a mutation each bit is flipped independently with some mutation probability. Using the reciprocal of the length of the bit strings as mutation probability, the most common and most recommended choice, on average in one mutation exactly one bit flips. This is similar to flipping exactly one bit chosen uniformly at random. We discuss the open problem of finding a useful characterization of fitness functions where replacing such local search by standard bit flip mutations does not lead to worse performance. We explain why this problem is interesting and important and how useful a solution would be. After this motivation we consider a few instructive example functions that help us understand where difficulties in solving this problem are.

Joint work of: Doerr, Benjamin; Jansen, Thomas; Klein, Christian

#### A Field Guide to Genetic Programming

William Langdon (University of Essex, GB)

A field guide to genetic programming, Riccardo Poli and William B. Langdon and Nicholas Freitag McPhee (With contributions by J. R. Koza), lulu.com

I would like to thank Dagstuhl and the organisers of Seminar 08051 "Theory of Evolutionary Algorithms" for hosting all three authors of "A field guide to genetic programming". Great progress was made during our week at the castle. (However this did mean one of us, WBL, was not able to formally present during the seminar). I hope you will be please to learn that the book is now finished.

A Field Guide to Genetic Programming is an introductory level text. Therefore the authors wanted it to be as widely available as possible. In practise, this means, as affordable as possible. Also, at the start of the twenty first century, this means it must be accessible via the world wide web. To both ends, we decided to make the whole book available as a freely down loadable PDF. The PDF makes wide use of hyperlinks, both internally and externally, via the genetic programming bibliography, to the literature cited. However anticipating some would still prefer to hold the text in their hands but be unwilling to print 250 pages, we opted to use an inexpensive print on demand service offered almost at cost by lulu.

The book sold out at its launch at EuroGP and down loads past 2000 copies in less than two weeks

Keywords: Genetic algorithms, genetic programming

Full Paper:

http://www.gp-field-guide.org.uk/

#### N-gram GP: Early Results and Half-Baked Ideas

Nicholas Freitag McPhee (University of Minnesota - Morris, USA)

In this talk I present N-gram GP, a system for evolving linear GP programs using an EDA style system to update the probabilities of different 3-grams (triplets) of instructions. I then pick apart some of the evolved programs in an effort to better understand the properties of this approach and identify ways that it might be extended.

Doing so reveals that there are frequently cases where the system needs two triples of the form ABC and ABD to solve the problem, but can only choose between them probabilistically in the EDA phase. I present the entirely untested idea of creating a new pseudo-instruction that is a duplicate of a key instruction. This could potentially allow the system to learn, for example, that AB is always followed by C, while AB' is always followed by D.

Keywords: Genetic programming, estimation of distribution algorithms, linear GP, machine learning

Joint work of: McPhee, Nicholas Freitag; Poli, Riccardo

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2008/1483

#### Weighted Multirecombination Evolution Strategy with Mutation Strength Self-Adaptation on Quadratic Sphere

Alexander Melkozerov (Fachhochschule Vorarlberg, A)

In the talk, a weighted multirecombination evolution strategy (ES) with mutation strength self-adaptation was presented along with the performance analysis of this ES on quadratic sphere. It was shown that the theoretical formula for calculation of optimal learning parameter allowed for tuning of the newly designed ES for maximal performance. An experimental comparison of the performance showed that the weighted multirecombination ES with mutation strength self-adaptation outperformed the weighted multirecombination ES with cumulative step size adaptation.

 $\begin{tabular}{ll} Keywords: & Weighted Multire$  $combination, Mutation Strength Self-adaptation, Evolution Strategy \\ \end{tabular}$ 

#### Estimating the Stationary Distributions of Markov Chains Modelling Evolutionary Algorithms Using Quotient Construction Method

Boris S. Mitavskiy (University of Sheffield, GB)

The evolutionary algorithm stochastic process is well-known to be Markovian. These have been under investigation in much of the theoretical evolutionary computing research. When mutation rate is positive, the Markov chain modelling an evolutionary algorithm is irreducible and, therefore, has a unique stationary distribution, yet, rather little is known about the stationary distribution. On the other hand, knowing the stationary distribution may provide some information about the expected times to hit optimum, assessment of the biases due to recombination and is of importance in population genetics to assess what's called a "genetic load" (see the introduction for more details). In this talk I will show how the quotient construction method can be exploited to derive rather explicit bounds on the ratios of the stationary distribution values of various subsets of the state space. In fact, some of the bounds obtained in the current work are expressed in terms of the parameters involved in all the three main stages of an evolutionary algorithm: namely selection, recombination and mutation. I will also discuss the newest developments which may allow for further improvements of the bounds

Keywords: Genetic algorithms, Markov chains, stationary distribution, lumping quotient

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1484

#### Making Problems Easier by Multi-Objective Optimization

Frank Neumann (MPI für Informatik - Saarbrücken, D)

Evolutionary algorithms have been shown to be good problem solvers for several optimization problems. Considering at least two objective functions that have to be optimized at the same time, we are dealing with multi-objective optimization problems. Often it is assumed that more objectives make a problem harder as the number of different trade-offs may increase with the number of objectives that are considered. In this talk, we examine how solving single-objective problems can be made easier by multi-objective optimization.

First, we consider the effect of adding objectives to a given problem. Experimental studies show that additional objectives may change the runtime behavior of evolutionary algorithms drastically. We illustrate the effect of adding objectives by considering simple functions and point out by theoretical analyses that this may be both beneficial and obstructive.

Later on, we consider the approximation ability of evolutionary algorithms for the class of covering problems and compare single-objective and multi-objective models for such problems.

We show that optimal solutions can be approximated within a factor of  $\log n$ , where n is the problem dimension, using a multi-objective approach while the approximation quality obtainable by a corresponding single-objective one may be arbitrarily bad.

#### Bye, Bye, Bloat

Riccardo Poli (University of Essex, GB)

During the last 15 years, intense research has been devoted to bloat (the growth in average program size without corresponding improvements in fitness) in genetic programming. Many theories have been proposed to explain this phenomenon. None has acquired general acceptance. The reason is probably that none was correct. Due to the lack of an explanation many empirical techniques to control bloat have been proposed. Only one has some theoretical foundation. Clearly, an unsatisfactory situation.

During the last 12 months or so, at long last, we (believe we) have found the real explanation for bloat. The explanation is so simple that it is almost unbelievable that it remained undiscovered for so long. In this talk I'll explain what it is, what evidence we have and what practical recipes and algorithms we have recently been able to devise to control bloat in a principled manner.

#### Monotonic Convergence and Memory Requirements of Algorithms for "Interactive" Search Problems

Elena Popovici (Icosystem, USA)

For traditional single-objective optimization problems, there exists a very weak criteria of monotonic algorithm convergence to a solution. This criteria is satisfied even by random sampling with constant memory (save just one best-so-far sample). I introduce an adaptation of this criteria for algorithms attempting to solve so-called "interactive" search problems. For such problems, it is non-trivial to construct algorithms that guarantee even this weak monotonic convergence with constant memory. For two types of interactive search problems, algorithms have been proposed that guarantee the weak monotonic convergence, but require unbounded memory in the worst case. I characterize in more detail these memory requirements. Finally, I investigate a third type of interactive search problem, for which no algorithms guaranteeing weak convergence have yet been proposed. I speculate that, for this problem type, even with unbounded memory, an algorithm could only provide the guarantee if it explored the space in one of two specific orders (both of which are, unfortunately, prohibitive in practice).

Keywords: Interactive search, monotonicity, convergence, memory

#### Solving Problems with Critical Variables

Adam Prügel-Bennett (Univ. of Southampton, GB)

We consider a class of problems with a small number of critical variables and many normal variables. The normal variables are easy to optimise, but their optimum value depend on the critical variables. The critical variables are only weakly coupled to each other, but strongly coupled with the normal variables. This leads to a set of problems with many local minima which are hard to solve with a hill-climbing algorithm. We show that a hybrid genetic algorithm can solve this class of problems vary efficiently. Furthermore, we will argue that many of the classic hard optimisation problems may have a structure which, at least approximately, is similar to the critical variable problems. We finally discuss some recent observation on the Max-Sat problem and argue that it may be possible to exploit information from a population to more effectively find good solutions.

 $Keywords\colon$  Critical variables, combinatorial optimisation, landscape structure, hybrid GA

## Representation-Invariant Crossover and Mutation Operators

Jonathan E. Rowe (University of Birmingham, GB)

The standard crossover and mutation operators on binary strings are invariant under certain syntactic changes in representation: for example, exchanging ones and zeros. I want to think about what this kind of invariance means for general search spaces, and characterise it using the idea of a group action. I then ask: how might we go about designing crossover and mutation operators for a search space that are guaranteed to be invariant under some choices of representation? A characterisation can be given in general terms, allowing the evolution equations for the infinite population model to be written in a canonical way for any (finite) search space, and providing an implementation method for such operators. However, a large open question remains: given a search space and an optimisation problem on it, what further properties do we want our genetic operators to have?

### Banach Space Techniques for the Analysis of Evolutionary Algorithms

Lothar M. Schmitt (The University of Aizu, J)

We study the steady-state probability distribution p for Markov chains G describing Genetic Algorithms (GA) and Genetic Programming (GP). That means: G p = p. Based upon a reduction method that associates any such Markov chain another Markov chain over 2 states only, we derive convergence properties for p in regard to uniform populations and populations containing globally optimal candidate solutions. Strong ergodicity of inhomogeneous Markov chains as above are discussed.

### Convergence in Co-Adapting Agents with Opponent Modelling

Jonathan L. Shapiro (Manchester University, GB)

We consider two co-adapting agents, each with a different and incomplete view of the environment and each trying to optimize a different objective function, which is a function of the behavior of of both agents. The agents learn to increase their objective function using gradient ascent, and learn the behaviour of their opponent using stochastic approximation. We show in the limit of an infinitesimal step size the following results: gradient ascent with perfect information of the opponent does not converge. It can be made to converge using the lagging anchor algorithm of Dahl. When the agents have to model each other's behaviour there is a phase transition between stable and unstable behavior

#### Runtime Analysis of Binary PSO

Dirk Sudholt (Technische Universität Dortmund, D)

We investigate the runtime of the Binary Particle Swarm Optimization (PSO) algorithm introduced by Kennedy and Eberhart (1997). The Binary PSO maintains a global best solution and a swarm of particles. Each particle consists of a current position, an own best position and a velocity vector used in a probabilistic process to update the particle's position. We present lower bounds for a broad class of implementations with swarms of polynomial size. To prove upper bounds, we transfer a fitness-level argument well-established for evolutionary algorithms (EAs) to PSO. This method is then applied to estimate the expected runtime on the class of unimodal functions. A simple variant of the Binary PSO is considered in more detail. The1-PSO only maintains one particle, hence own best and global best solutions coincide. Despite its simplicity, the 1-PSO is surprisingly efficient.

A detailed analysis for the function Onemax shows that the 1-PSO is competitive to EAs.

Keywords: Particle swarm optimization, runtime analysis

Joint work of: Sudholt, Dirk; Witt, Carsten

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1480

#### Complexity Lower Bounds for Evolution Strategies

Olivier Teytaud (Université Paris Sud, F)

Jens Jägersküpper has provided important lower bounds for evolution strategies in continuous domains. Teytaud and Gelly have provided some extensions using combinatorial arguments and in particular the branching factor of comparison-based evolution strategies.

Using geometric techniques and Sauer's lemma for level sets of finite VC-dimension, we show new bounds for (mu,lambda)-ES with application to the speed-up of parallel evolution strategies and derive practical hints for designing new mutation operators.

Keywords: Sauer's lemma, VC-dimension, complexity lower bounds

Joint work of: Teytaud, Olivier; Fournier, Hervé

#### Tight Bounds for Blind Search on the Integers

Ingo Wegener (Technische Universität Dortmund, D)

We analyze a simple random process in which a token is moved in the interval  $A = \{0, n\}$ : Fix a probability distribution  $\mu$  over  $\{1, \ldots, n\}$ .

Initially, the token is placed in a random position in A. In round t, a random value d is chosen according to  $\mu$ . If the token is in position  $a \geq d$ , then it is moved to position a-d. Otherwise it stays put. Let T be the number of rounds until the token reaches position 0. We show tight bounds for the expectation of T for the optimal distribution  $\mu$ , i.e., we show that  $min_{\mu}\{E\mu(T)\} = \Theta((logn)^2)$ . For the proof, a novel potential function argument is introduced. The research is motivated by the problem of approximating the minimum of a continuous function over [0, 1] with a 'blind' optimization strategy.

Joint work of: Dietzfelbinger, Martin; Rowe, Jonathan E.; Wegener, Ingo; Woelfel, Philipp

See also: Symposium on Theoretical Aspects of Computer Science 2008 (Bordeaux), pp. 241-252

#### Understanding Problem Structure and NFL

L. Darrell Whitley (Colorado State University, USA)

Two ideas are presented.

First, new proofs show that a subset of algorithms can have identical performance over a subset of functions, even when the subset of functions is not closed under permutation.

We refer to these as focused sets. In some cases focused sets correspond to the orbit of a permutation group; in other cases, the focused sets must be computed heuristically. In the smallest case, two algorithms can have identical performance over just two functions in a focused set. These results particularly exploit the case where search is limited to m steps, where m is significantly smaller than the size of the search space. These proofs emphasize the need for search algorithms to understand and exploit problem structure.

Second, the problem structure of elementary landscapes is reviewed, particularly for the Travelling Salesman Problem. These results suggest that current algorithms may not adequately (or at least explicitly) exploiting all of the available problem structure.

### Conditions for the Robustness of Compositional Coevolution

R. Paul Wiegand (University of Central Florida, Orlando, USA)

Analyses of compositional coevolution indicate collaboration methods seriously impact the optimization goals of the methods. Experience suggests compositional coevolutionary algorithms that use averaging mechanisms for collaboration (e.g., complete mixing) perform well in multiagent learning settings, where the goal is often to find team behaviors that are not necessarily optimal, but robust to

certain types of changes. At the 2006 Dagstuhl seminar, I introduced a new framework for rigorously defining robustness. In this year's talk, I extend this idea by introducing the notion of a parametric degree of robustness, and I demonstrate a particular condition that ensures progress in a dynamical systems model of the algorithm. The condition relates this parametric degree of robustness with the relative sizes of portions of the phase space of the modelled algorithm. In this way, one should be able to show that the larger the degree of robustness, the larger the basin of attraction for a particular solution: robust solutions are attractive to certain kinds of CCEAs.

Keywords: Compositional Coevolution, Robustness, dynamical systems, game theory

### Does a Temporally or Spatially Varying Environment Speed Up Evolution?

Alden Wright (Univ. of Montana - Missoula, USA)

Kashtan, Noor, and Alon (PNAS v. 104, 13711-13716) claim that for some problems, temporal variation among multiple goals speeds up evolution to a fixed goal. I have partially replicated their results, and have done preliminary tests on whether spatially varying goals speeds up evolution.

Keywords: Evolution, genetic algorithms, speedup

### Approximating the Pareto Set Using Set Preference Relations: A New Perspective On Evolutionary Multiobjective Optimization

Eckart Zitzler (ETH Zürich, CH)

Assuming that evolutionary multiobjective optimization (EMO) mainly deals with set problems, one can identify three core questions in this area of research: (i) how to formalize what type of Pareto set approximation is sought, (ii) how to use this information within an algorithm to efficiently search for a good Pareto set approximation, and (iii) how to compare the Pareto set approximations generated by different optimizers with respect to the formalized optimization goal. There is a vast amount of studies addressing these issues from different angles, but so far only few studies can be found that consider all questions under one roof.

This talk is an attempt to summarize recent developments in the EMO field within a unifying theory of set-based multiobjective search. It discusses how preference relations on sets can be formally defined, gives examples for selected user preferences, and proposes a general, preference-independent hill climber for

multiobjective optimization with theoretical convergence properties. The proposed methodology brings together preference articulation, algorithm design, and performance assessment under one framework and thereby opens up a new perspective on EMO.

 ${\it Keywords:}$  Evolutionary algorithms, multiobjective optimization, preference articulation

Joint work of: Zitzler, Eckart; Thiele, Lothar