# Executive Summary of Dagstuhl Seminar on Structured Decompositions and Efficient Algorithms (08492)


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## 1 Summary

New emerging technologies such as high-precision sensors or new MRI machines drive us towards a challenging quest for new, more effective, and more daring mathematical models and algorithms. Therefore, in the last few years researchers have started to investigate different methods to efficiently represent or extract relevant information from complex, high dimensional and/or multimodal data. Efficiently in this context means a representation that is linked to the features or characteristics of interest, thereby typically providing a sparse expansion of such. Besides the construction of new and advanced ansatz systems the central question is how to design algorithms that are able to treat complex and high dimensional data and that efficiently perform a suitable approximation of the signal. One of the main challenges is to design new sparse approximation algorithms that would ideally combine, with an adjustable tradeoff, two properties: a provably good 'quality' of the resulting decomposition under mild assumptions on the analyzed sparse signal, and numerically efficient design.

The topic is driven by applications as well as by theoretical questions. Therefore, the aim of this seminar was to bring together a good mixture of scientists with different backgrounds in order to discuss recent progress as well as new challenging perspectives. In particular, it was intended to strengthen the interaction of mathematicians and computer scientists.

The goals of the seminar can be summarized as follows:


Dagstuhl Seminar Proceedings 08492
Structured Decompositions and Efficient Algorithms
http://drops.dagstuhl.de/opus/volltexte/2009/1885

– Initiate communications between different focuses of research.
– Comparison of methods.
– Open new areas of applications.
– Manifest the future direction of the field.

## 2 Seminar Topics

To reach the seminar goals, the organizers identified in advance the most relevant fields of research:

– Geometric multiscale analysis.
– Compressed sensing.
– Frame construction and coorbit-theory.
– Sparse and efficient reconstruction.
– Probabilistic models and dictionary learning.

The seminar was mainly centered around these topics, and the talks and discussion groups were clustered accordingly. During the seminar, it has turned out that in particular 'compressed sensing' and 'sparse and efficient reconstructions' are currently the most important topics. Indeed, most of the proposed talks were concerned with these two issues. This finding was also manifested by the discussion groups. Although originally small discussion groups for all the above topics were scheduled, the interest was absolutely centered around 'compressed sensing' and 'sparse approximation', so that only these two topics were discussed in greater details in small groups. For a detailed description of the outcome of the results, we refer to Section 3.

The course of the seminar gave the impression that *sparsity* with all its facets will probably be one of the most important techniques in applied mathematics and computer sciences in the near future. It was fascinating to see how sparsity concepts are by now influencing many different fields of applications ranging from image processing/compression to adaptive numerical schemes and the treatment of inverse problems. It seems that the sparsity concept allows to exploit a very similar fundamental structure behind all these different applications. Closely related with sparsity is the concept of *compressed sensing*. Originally developed as a tool for sparse signal recovery, ideas from compressed sensing are now flowing into related fields such as numerical analysis and provide a way to design, e.g., more efficient adaptive schemes.

During the seminar, the feeling emerged that in the future the most important progress will probably be made on an *algorithmic* level. The development of new building blocks by itself seems to be of limited use. Instead, more or less data driven algorithms are needed. Moreover, most of the participants had the impression that the time is ripe to deal with new and challenging applications for the following reason. It seems that in many classical fields of applications such as denoising and compression the mathematical methods have almost reached the ceiling and important further progress cannot be expected. Nevertheless, there is an urgent need to derive efficient algorithms for complex, i.e., high-dimensional and/or multimodal data etc., and these problems should be energetically attacked. In doing so, in particular *geometric* information should be exploited. On an algorithmic level, more effort should be spend to take into account the geometric structures of the signals to be analyzed, to efficiently detect lower-dimensional manifolds on which the essential information is concentrated etc. Moroever, we need a deeper geometric understanding of dictionaries. The choice of the 'best dictonary' with respect to the overall sparsity criteria seems to be essential, and for this, concepts from dictionary learning will probably be very important.

## 3 Outcome of the Discussion Groups

As already mentioned, the seminar interest was mainly centered around the topics 'compressed sensing' and 'sparse approximation' and therefore only two discussion groups took place.

- The discussion on 'compressed sensing' (chaired by W. Dahmen) started collecting the following guiding questions and issues:

  - Possible further developments of sparsity models for analog signals and corresponding formulations that can be treated by compressed sensing techniques.
  - A second complex of questions centered around the precise meaning of compressed sensing and resulting limitations. This concerns, in particular, the possibility of progressively upgrading resolution.
  - The conceptual scope of applications of compressed sensing and its potential role in other application areas.
  - The construction of deterministic sensor matrices.

  The first part of the discussion was primarily around construction principles of deterministic sensor matrices, which was seen as a question of central importance. All currently known attempts appear to be based on the concept of coherence which significantly limits the feasible sparsity range to the square root of the number of measurements. A specific ansatz for a concrete sensing matrix was discussed in greater detail, requiring at some point sharp estimates for multiple Gauss sums that may bring in number theoretic aspects. Another direction (but still random) was a construction principle of matrices with less demanding properties than RIP that still give rise to meaningful estimates for coefficients of the signal that are substantial compared with the energy of the whole signal. The remaining discussion revolved around questions concerning compressed sensing techniques for analog signals. This direction of research is only beginning now and the currently used models should be seen as first examples.

In summary, the general agreement is that compressed sensing has brought in starting new perspectives stressing in particular the beneficial effect of randomness that calls for further investigations and efforts concerning practical applications as well as conceptual clarification regarding its scope and bearing on other areas. One of the challenging questions may concern a proper embedding of such concepts into an infinite dimensional framework.

- The discussion on ‘sparse approximation’ (chaired by Alfred Bruckstein) started by an attempt to define the state-of-the-art in this field, as a stepping stone for defining future challenges. The ingredients of the state-of-the-art the discussion brought up include

  - Sparsity – an established concept with widespread use and understanding. This is the major achievement that drives the rest.
  - Solid mathematical results in this field that add to its beauty and maturity. Specifically, proofs for the successful behavior of pursuit algorithms of various sorts to solve what seems as NP-hard problems.
  - Advances in numerical tools that support the practicality of this field. Those include iterative shrinkage algorithms for practical pursuit.
  - Success stories in applications in signal processing, image processing, and other fields.
  - Emergence of new dictionaries that serve the sparsity model better, and thus enable state-of-the-art performance in various applications.

  Armed with the above definition of the state-of-the-art, the discussion then turned to future work. First, considering where this field is going next, the following core points were mentioned and discussed:

  - Towards new dictionary design procedures, and ways to incorporate structure into them.
  - Deeper geometric understanding of good dictionaries in an attempt to bridge between the new area of dictionaries and past-used frames.
  - More analytical results (extensions of the theoretical analysis of pursuit methods mentioned above), defining new and related problems.
  - Nonlinear and adaptive designed dictionaries, and ways to extend the sparsity model to realistic data.

  The final part in the discussion opened the stage for the dreams the participants have in terms of their research goals and desires. Topics mentioned include:

  - Geometry and its relation to Gestalt.
  - Operators and function spaces, and using these connections to obtain good dictionaries.
  - 3D and beyond for representation systems.

- Divide and conquer and online algorithms.
- Inverse problems and recognition.
- Bilateral and nonlinear connection between coefficients.
- More probabilistic approaches/arguments/analysis.
- More nonlinearity and adaptivity.
- Size of dictionary issues.
- What is the correct way to measure errors in representation?

## 4 Final Concluding Remarks

This seminar was regarded by the participants as a very productive and inspiring meeting. Many intense discussions took place throughout the week, and several new cooperations were initiated. Especially, the interactions between computer scientists and applied mathematicians has been extremely fruitful and will certainly be continued in the future. Also, the major future directions of this research area were manifested and initial steps towards solutions undertaken. Concluding, this seminar can be regarded as a milestone in the development of the new, rapidly evolving research area of Structured Decompositions and Efficient Algorithms.

Last, but not least, the success of this seminar is in main parts due to the great scientific atmosphere offered by Schloss Dagstuhl, for which we would like to thank the scientific as well as administrative staff at Schloss Dagstuhl.